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EVOLUTION OF TEMPLE ELEVATIONAL FORM WITH SQUARE CIRCLE METHOD: LAKSHMAN TEMPLE IN SIRPUR

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Abstract

One of the fundamental methods for shaping the constructional geometry of any building is the use of basic shapes: circles and squares. The circle represents vitality or energy, while the square represents strength. In world history, the concept of geometry traces its origins to construction in Egypt and Babylonia, where proportional systems were described through mathematical equations. They later became known as the Pythagorean Theorem, named after Pythagoras. In Ancient India, the concept of geometry starts with the construction of *altars* for *Vedic* sacrifices, as per the instructions of the *Śulbasūtras*. This involved creating circles and squares, converting squares to circles and vice versa, resulting in altars of various shapes and proportionate systems. The intersection of these basic shapes, the square and the circle, is the key to constructional building geometry. For instance, *Vesica* Piscis is a geometrical element derived from the circle-circle intersection. It has been applied by researchers to examine the geometry of both ancient and modern buildings. Similarly, the Square-Circle Sequence (SCS) is a method derived from the square-circle intersection. Gandotra (2011) used it to study the constructional geometry of the Hindu temples in North India (*Nāgara temples*). Meister (1985) also applied the square-circle intersection geometric constructional method to define the proportionate system of the Hindu temples in India. Finally, this study attempts to correlate these types of constructional geometry in the evolution of elevational form of *Nāgara temples* through *Lakshman temple in Sirpur*. It determines that the building's elevational form may be derived from the basic shapes of the circle and the square.

Keywords: Elevational form, Śulbasūtras, Vesica Piscis, square-circle sequence, circle-circle intersection, square-circle intersection, Lakshman Temple in Sirpur.

Introduction

Early in history, humans built their huts and erected their tents with an intuitive notion of geometry. Basic ideas of shape and form could have emerged from observing the sky and nature, and could then have been developed further for practical needs, such as measuring and calculating area sizes (Srinivasan, 2010). Thus, geometry (where "geo" means "earth" and "metron" means "measurement", as per ancient Greek etymology) is a branch of mathematics that deals with shape, form and measurement, with the visual component being dominant. To create geometric shapes, figures or areas need to be enclosed by a boundary that consists of a specific amount of curves, points and lines. The most common, basic geometric shapes are the circle and various polygons.

The origins of geometry go back to the insights obtained with certain mathematical equations and formulas in Ancient Greece, India, Egypt, Babylonia, and China. The difference in approach between these civilizations can be illustrated by how they would solve for x in this equation: $x^2 = N$ (the concept of the square root) (Joseph, 1997).

The geometric principles expounded in the *Śulbasūtras* (800–500 BCE) have often been considered the beginning of mathematics on the Indian subcontinent (Sinha et al., 2011). The term *"śulba"* or *"śulva"* comes from the root *"śulv"*, which can be a verb, "to measure" (Harding, 2004), or a noun, "string, cord or rope" (Price, 2000). Vedic literature has forty parts: the four Vedas plus six additional sections, consisting of six parts each. These sections are the Vedas, the Vedāṅgas, the Upāṅgas, the Upa-Vedas, the Brāhmaṇas, and the Prātishākhyas. The *Śulbasūtras* form part of the Kalpa Sūtras, which, in turn, are part of the Vedāṅgas. There are four main *Śulbasūtras*: the Baudhāyana, the Āpastamba, the Mānava, and the Kātyāyana (Price, 2000). The *Śulbasūtras* describe the construction of altars of various shapes, depending on the particular ritual (Seidenberg, 1961): specifically, square and circular altars for domestic sacrifices, and other shapes like śyena citi (falcon-shaped), rathachakra citi, and kūrma citi for special sacrifices.

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Using squares and circles in construction as per the *Śulbasūtras*

The construction of citis (altars or ceremonial platforms) starts with geometrical and arithmetical calculations and ends up with detailing. Geometrical measurements are performed by drawing arcs with different radii and centers using a cord, or *śulba* (Price, 2000).

The procedure starts with drawing the *prācī*, which is a line in the east-west direction (Price, 2000). According to ancient Indian texts (Mayamata and Mānasāra), directions were determined with the gnomon method. In this method, an umbrella-shaped gnomon — measuring 24, 18 or 12 aṅgulas (fingers) in length; 6, 5 or 4 aṅgulas (respectively) at the base; and 2, 1 or ¾ aṅgulas (respectively) at the top — is installed on the selected area and leveled with water. A circle (defined as a shape that is created by a set of points in a plane that are equally distant from a fixed point, i.e., the center) is drawn starting from the gnomon's bottom, with a radius twice the length of the gnomon. Two points are marked on the circumference of the circle when the shadow of the gnomon passes through them, before and after noon. The straight line joining these two points is roughly taken to be the east-west line (*prācī*). The line that bisects the east-west line is, therefore, the north-south line. The bisecting is done in the usual manner. Two circles are drawn, with the respective ends of the east-west line functioning as their centers and the length equaling their radius. They intersect at two points, creating a Vesica Piscis shape; the straight line joining the points of intersection bisects the east-west line at right angles and indicates the north-south line. The intermediate quarters are outlined in the same manner: by constructing the Vesica Piscis between the points of the previous sections (Acharya,1994; Dagens, 1985) (Fig. 1).

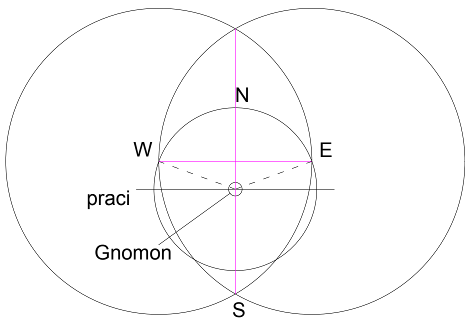

Fig. 1. The gnomon method as per Mayamata and Mānasāra, used for identifying the site's orientation (drawn by the author)

After the *prācī* is laid down EOW, the circles are used to create a square; the steps are shown in Fig. 2.

Step 1 — The *prācī* is drawn EOW, and the intersection of two circles, with radii equaling the length of the *prācī*, gives us the north and south directions.

Step 2 — New circles are drawn in a similar manner in all directions, all with the same radius (OE) and with the E as the center.

Step 3 — The intersection point of two circles at the east and north direction gives us the north-east direction (A). The respective intersection points in the other directions are B, C, and D, producing the required square.

Verse I, 48 of Baudhāyana *Śulbasūtras* states, "The diagonal of a rectangle produces both areas that its length and breadth produce separately. This is seen in rectangles with sides three and four, twelve and five, fifteen and eight, seven and twenty-four, twelve and thirty-five, fifteen and thirty-six". The statement is directly related to the Theorem of Pythagoras, and the numbers it mentions are the "Pythagorean Triples", which the ancient Indians used for constructing various right angles.

Verse I, 50 of Baudhāyana *Śulbasūtras* says that a square is equal to the sum of the two unequal squares. The geometrical construction is described in Fig. 3. And verse I, 51 of Baudhāyana *Śulbasūtras* mentions that a square is also equal to the difference between two unequal squares. This geometrical construction is described in Fig. 4 (Price, 2000).

In verse I, 52 of Baudhāyana *Śulbasūtras,* the diagonal of a square is described as the side of another square with two times the area (Fig. 5). Let us suppose that the side of a square is one unit; then the diagonal is its square root (√2), or *dvi-karaṇī* (meaning "that which produces 2", or "double-

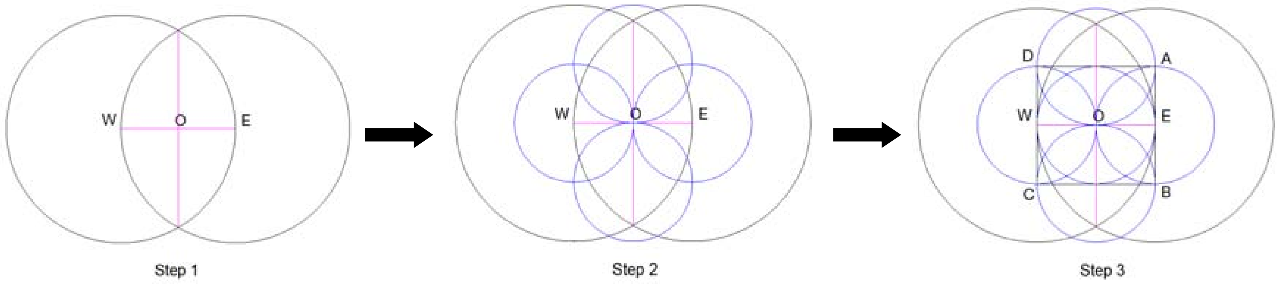

Fig. 2. Step-by-step process of constructing a square from the given *prācī* (drawn by the author)

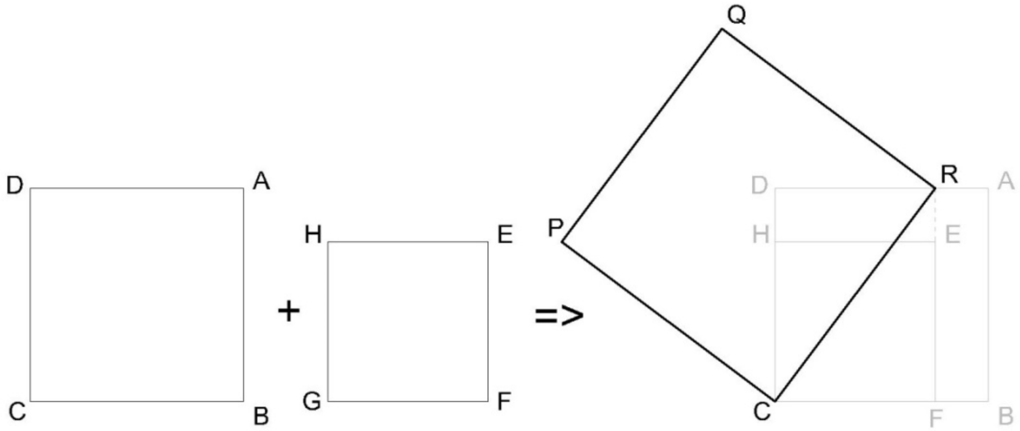

Fig. 3. Sum of two unequal squares. ABCD > EFGH. Adding the two squares leads to DR = CF = GF; then CR is the side of the square whose area is equal to the areas of ABCD and EFGH (drawn by the author)

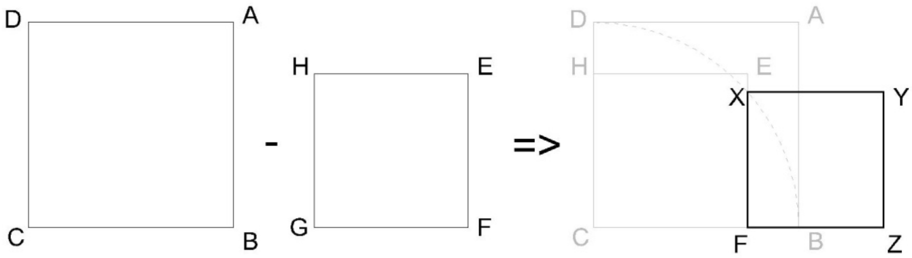

Fig. 4. Difference between two unequal squares. ABCD> EFGH, $CX^2 = CF^2 + XF^2$. If CB = CX, then $FX^2 = CB^2 – CF^2$, where FX is the side of the square whose area is equal to the difference between the areas of ABCD and EFGH (drawn by the author)

maker”) (Henderson, 2000). If a rectangle is then formed with the proportion of one to the square root of two, its diagonal would be *tri-karaṇī* (“that which produces 3”, or “triple-maker”) (Harding, 2004).

The length of the diagonal in a square with a one-unit side is √2 = 1.4142135… (as per the calculator). Verses I, 61-2 of Baudhāyana *Śulbasūtras*, I, 6 of Āpastamba *Śulbasūtras* and II, 13 of Kātyāyana *Śulbasūtras* state the following: “Increase the length of the side by its third, and this third by its own fourth less the thirty-fourth part of that fourth”. Arithmetically this is expressed as:

$$\sqrt{2} = 1+\frac{1}{3}+\frac{1}{4}\cdot\frac{1}{3}-\frac{1}{34}\cdot\frac{1}{4}\cdot\frac{1}{3}.$$

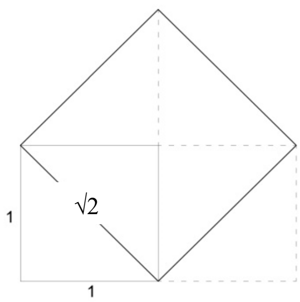

Fig. 5. *Dvi-karaṇī* (drawn by the author)

The value is √2 = 1.4142156…, which is very close to √2 = 1.4142135… (as per the calculator). The geometrical construction corresponding to the √2 calculations described in the *Śulbasūtras* is shown in Fig. 6.

Verse I, 54 of Baudhāyana *Śulbasūtras* says, “If you wish to turn a rectangle into a square, use the shorter side of the rectangle as the side of the square, divide the remaining two parts, invert them, and join them into two sides of the square”. This geometrical construction is explained in Fig. 7.

Thus, the *Śulbasūtras* provide the geometrical and arithmetical solution for using the fundamental shapes of the circle and square in construction, and also give an idea of how to convert a rectangle into a square. The square root concepts can be derived from the sūtras as well.

Converting a square into a circle

Verse I, 58 of Baudhāyana *Śulbasūtras* states, “If one desires to transform a square into a circle, (a cord of length) half the diagonal (of the square) is stretched from the center to the east (with a part of it lying outside the eastern side of the square); with one-third (of the part lying outside) added to the remainder (of the half diagonal), the (required) circle is drawn” (Sen and Bag, 1983).

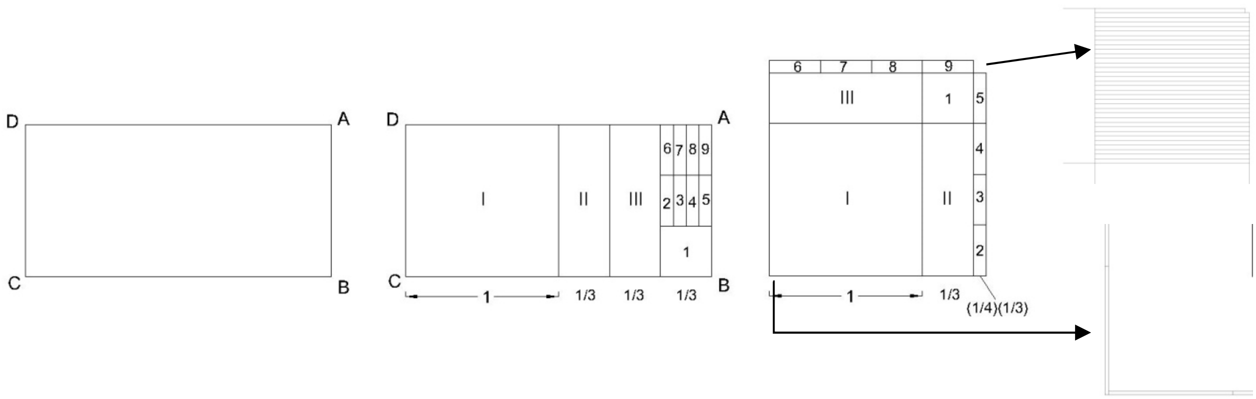

Fig. 6. Deriving the value of √2 (drawn by the author)

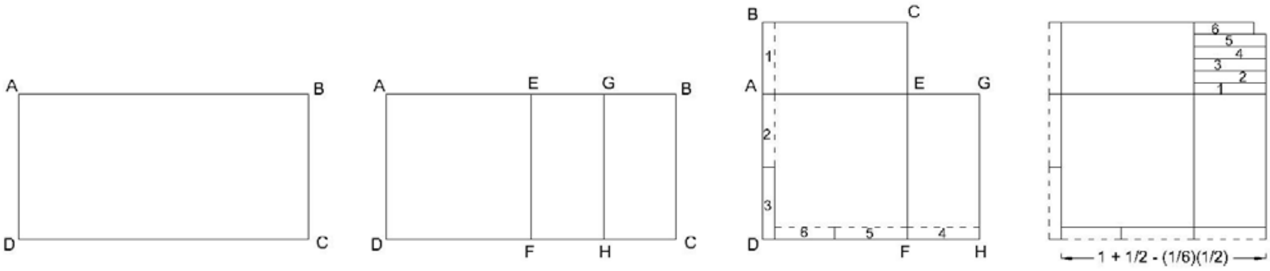

Fig. 7. Converting a rectangle into a square (drawn by the author)

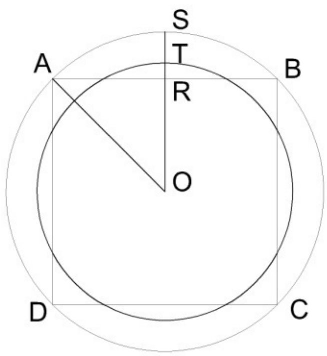

Fig. 8. Converting a square into a circle (drawn by the author)

Fig. 8 demonstrates how this is done:

Let ABCD be a square with a center in point O. Draw an arc, AS, with its center being O and radius being OA, so that OS is parallel to AD. Let us suppose OS intersects AB at point R.

Let T be the point on RS at 1/3 of the distance from R to S. Then OT is the radius of the required circle.

Let 2*a* be the side of the square and *r* be the radius of the constructed circle. Then:

$$\boldsymbol{r} = \boldsymbol{OT} = \boldsymbol{OR} + \boldsymbol{RT} = \boldsymbol{OR} + \tfrac{1}{3}\left(\boldsymbol{OS} - \boldsymbol{OR}\right) =$$

$$= \boldsymbol{a} + \tfrac{1}{3}\left(\boldsymbol{a}\sqrt{2} - \boldsymbol{a}\right).$$

Hence:

$$\boldsymbol{r} = \tfrac{\boldsymbol{a}}{3}\left(2 + \sqrt{2}\right).$$

Now, the area of the constructed circle is as follows:

$$\boldsymbol{Area} = \pi \boldsymbol{r}^2 = \pi\left(\tfrac{\boldsymbol{a}}{3}\left(2 + \sqrt{2}\right)\right)^2.$$

If we substitute the value of π = 3.141593 and $\sqrt{2}$ = 1.414214, we get the area of the constructed circle:

$$\boldsymbol{Area} = \pi \boldsymbol{r}^2 = 4.069009\, \boldsymbol{a}^2,$$

which is within about 1.7 % of the correct value for the area of the square = 4 (Price, 2000).

Converting a circle into a square

To continue, verse I, 59 of Baudhāyana *Śulbasūtras* states that to transform a circle into a square, the diameter is divided into eight parts; one such part is also divided into twenty-nine parts, then twenty-eight of them are removed, and the last part left is further reduced by one sixth and then one eighth (of the sixth) (Sen and Bag, 1983)

This equation can be solved as follows:

Let the diameter of the circle be *d*, then the length (*a*) of the required square is as follows:

$$\boldsymbol{length\ a} = \boldsymbol{d} - \frac{\boldsymbol{d}}{8} + \frac{\boldsymbol{d}}{8 \times 29} - \frac{\boldsymbol{d}}{8 \times 29}\left(\frac{1}{6} - \frac{1}{6 \times 8}\right)$$

$$= \frac{9785}{11136}\boldsymbol{d}.$$

Now, the area of the square = the area of the circle.

Therefore: $\dfrac{\boldsymbol{Area\ of\ the\ square}}{\boldsymbol{Area\ of\ the\ circle}} = \dfrac{\boldsymbol{a}^2}{\pi \boldsymbol{r}^2} =$

$$= \frac{\left(\frac{9785}{11136}\right)^2 d^2}{\frac{\pi}{4} d^2} = = 0.983045.$$

The ratio should be 1, and so the result is accurate with a margin of approximately 1.7%, same as in the previous section (Price, 2000).

Circle-circle intersection geometry

According to Euclidean geometry, to construct an equilateral triangle with a line segment as one of the sides (Fig. 9), we need to do the following:

Let AB be the finite line segment. Then we draw a circle (D), with its center being in point A and radius being AB. Similarly, another circle (E) is drawn with its center in point B. Let the two circles intersect at point C. These points are joined to form lines AC and BC. This results in an equilateral triangle, ABC.

The following proves that the constructed triangle is an equilateral triangle:

A is the center of circle CDB.

Thus, AC = AB ---------------- 1

B is the center of circle ACE.

Thus, AB = BC ------------------ 2

From 1 and 2, we conclude that AC = AB = BC.

Therefore, ABC is an equilateral triangle (Srinivasan, 2010).

Thus, the simple, complex, and advanced geometrical shapes can be derived from points, lines, and planes with the help of advanced techniques.

When any two circles intersect, this produces an almond shape, but when two circles of identical size intersect in such a way that the center of one lies on the circumference of the other, this produces Vesica Piscis (Fletcher, 2004). Vesica Piscis is a Latin term, where *Vesica* means “bladder” and *Piscis* means “fish” (Fletcher, 2004). Barrallo et el. (2015) stated that this geometric form can be expanded to construct two contiguous equilateral triangles with opposite orientation in the intersection between the circles. From this, it can be easily demonstrated that the ratio between the vertical and horizontal proportions of Vesica Piscis is the square root of various proportional systems (Fletcher, 2004).

Fletcher (2004) mentioned that Vesica Piscis can also signify the womb — in Christianity, that would be the womb of the Virgin from which Christ emerges. Additionally, Vesica Piscis proportions appear in the Gothic arch and underlie rectangular floor plans of numerous churches and chapels (Fletcher, 2004).

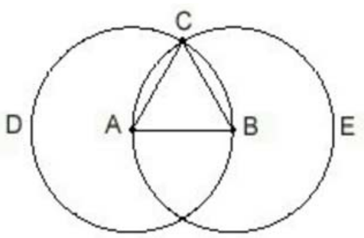

Fig. 9. An example of a metrical theorem (Victor Katz, A History of Mathematics: An Introduction, 1998) (Srinivasan, 2010)

Proportional systems in Vesica Piscis

Starting from the two circumferences, γ1 and γ2, we want to compute the lengths of AB and CD for the respective segments. Let r1 and r2 denote, respectively, the radii of the γ1 and γ2 circumferences. We thus have r1 = r2 = 1. Segment AB is, by construction, the radius of both circumferences, and its length equals one unit. Since O is the midpoint of segment AB, segment AO has a length of 1/2 unit. Similarly, by construction, AC is the radius of circumference γ1, and its length equals 1. This results in an equilateral triangle, ABC. Triangle AOC is rectangular, with segment CO being perpendicular to AB (Sparavigna and Baldi, 2016).

a. Equilateral triangle and the ratio of √3:1

Therefore, we can apply Pythagoras’ Theorem to triangle AOC (Figs. 10 and 11) and calculate the length of segment CO. We obtain the following:

$$CO = \sqrt{(AC^2 - AO^2)} = \sqrt{[1^2 - (1/2)^2]} = \sqrt{3}/2. \quad (1)$$

According to the symmetry in Fig. 11:

$$CD = 2CO = 2\left(\sqrt{3}/2\right) = \sqrt{3}. \quad (2)$$

The ratio of the two main segments is:

$$CD/AB = \sqrt{3}/1 = \sqrt{3}. \quad (3)$$

We thus have geometrically obtained the square root of 3, which is an irrational number. However, since we referred to Pythagoras’ Theorem, let us consider Eq. (3), written as follows:

$$CD^2/AB^2 = 3. \quad (4)$$

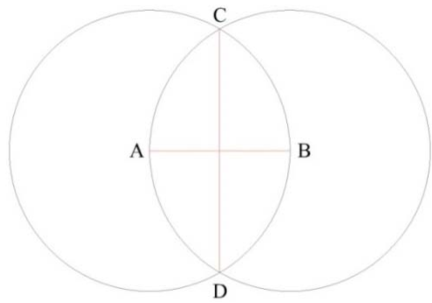

Fig. 10. CD:AB : √3:1 (drawn by the author)

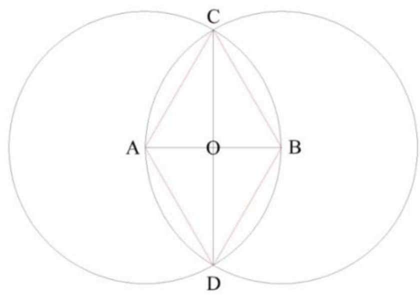

Fig. 11. OC:OB : √3:1 (drawn by the author)

b. Square and the ratio of √2:1

We draw two circles with radius AB = 1 (Fig. 12).

We then locate the vertical diameters, EF and GH, of the generating circles (EF = GH = 2).

We connect points ABGE and ABHF. This results in two squares.

We draw diagonal BE through square ABGE.

Thus, by applying Pythagoras' Theorem to triangle ABE, we have

$$EB^2 = \sqrt{\left(AB^2 + AE^2\right)} = \sqrt{\left(1^2 + 1^2\right)} = \sqrt{2}. \quad (5)$$

Similarly, we connect points EFHG (Fig. 13), to create rectangle EFHG.

We then draw diagonal EH through rectangle EFHG.

Now, by applying Pythagoras' Theorem to triangle FHE, we obtain:

$$EH^2 = \sqrt{\left(FH^2 + EF^2\right)} = \sqrt{\left(1^2 + 2^2\right)} = \sqrt{5}. \quad (6)$$

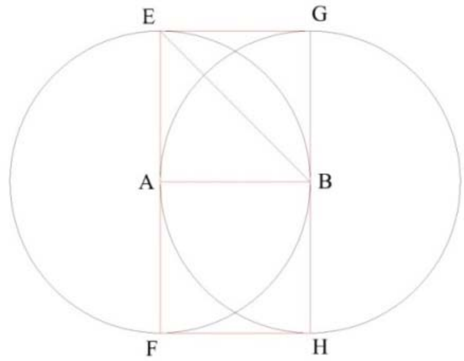

Fig. 12. BE:AB : √2:1 (drawn by the author)

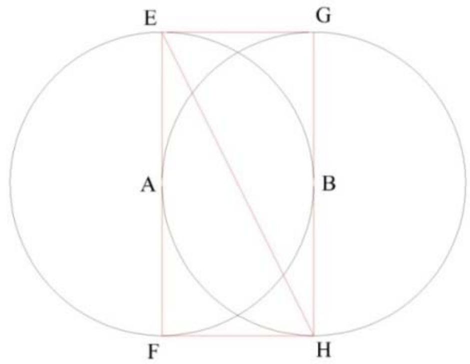

Fig. 13. EH:HF : √5:1 (drawn by the author)

Circle-circle intersection geometry in architecture

Barrallo et al. (2015) mentioned that during the Gothic period, many structural and ornamental elements utilized a geometry based on circle-circle intersections: the arches, windows, vaults, porches, and traceries that form the distinctive features of Gothic cathedrals contain Vesica Piscis forms.

In modern architecture, Vesica Piscis, or circle-circle intersection geometry, can be seen in the designs and works of Norman Foster and Santiago Calatrava. Repeatedly using the Vesica Piscis construction technique, with spheres spread out across three dimensions, is a way to create a complex building design. Probably the most important building of this category is the Sydney Opera House, designed by the Danish architect Jørn Utzon in 1957 (Barrallo et al., 2015). The Lotus Temple (Bahai house of worship) in Delhi, designed by the architect Fariborz Sahba in 1986, can also be categorized under the Vesica Piscis complex construction technique.

Square-circle intersection

Baudhāyana *Śulbasūtras* provide the geometrical methods for constructing altars in rectilinear shapes and converting them into other shapes. Thus, the square and the circle are considered to be the fundamental shapes for constructing any geometrical figures. They can also be adopted as geometrical tools for determining the various geometries in a building.

The *Bṛhat Saṁhitā* part of Varāhmihira (dating back to 6th century AD) says in chapter LIII, "House Building" that residential buildings are to be built in a square grid of 9 x 9 = 81 squares and that temple buildings are to be built in a square grid of 8 x 8 = 64 squares, as per the *Vāstūpūrūshamaṇdala*. The *Mayamata* by Dagens (1985) records thirty-two types of square *maṇdala*, which create *maṇdala* arrangements suitable for all construction sites. These range from *sakala*, consisting of just one square, to *indrakānta*, consisting of 1024 squares. Rian *et al.* (2007) adopted two types of square *maṇdala* — *yugma maṇdala* (even number of square grids) and *ayugma maṇdala* (odd number of square grids) — to analyze the constructional planning of the Kandariya Mahadev Temple in Khajuraho, which is a *fractal iteration of maṇdala (*Fig. 14).

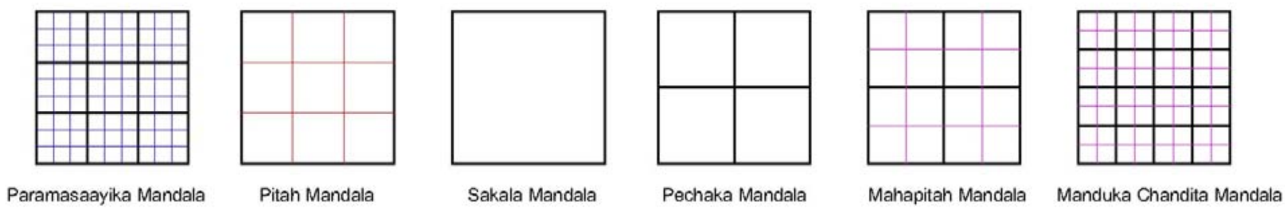

Fig. 14. Fractal iteration of *maṇdala* (Rian *et al.* 2007) (drawn by the author)

The inherent geometry of Indian temples, governing their planning and overall form, has been studied and documented widely by researchers. One of the methodologies developed by Gandotra (2011) is the *Square-Circle Sequence* (SCS). The method is based on a simple sequence of squares and circles (Gandotra, 2011). The author explained that "the squares are rotated at 22.5º to form a 16-point star (the 16 points can be equated to the 16 petals of the *Sri Yantra*) (Fig. 15).

The intersection points (*marmas*) of this 16-point figure, when projected, provide all the key locations of the temple plan". Notably, depending on the deity in the temple's sanctuary (*garbhagriha*), the SCS starts with either a circle (if the deity is a *linga*) or a square (if the deity is on a pedestal) (Fig. 16).

Proportional system of the SCS

The SCS is a generic figure that will have the following sequence for its side of the square: 1x, 1.4x, 2x, 2.8x, 4x, 5.6x, 8x, 11.3x, 16x… (Gandotra, 2011). We can see that every alternate dimension is doubling.

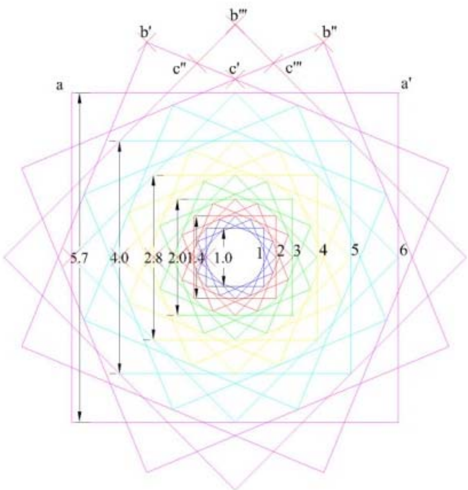

Fig. 15. Constructing a Square-Circle Sequence (SCS) to form a 16-point star (*mamra*) (drawn by the author)

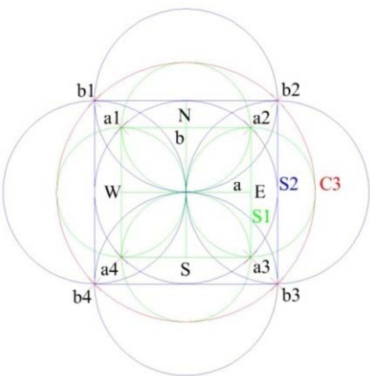

Fig. 16. Development of the Square-Circle Sequence (SCS) diagram (drawn by the author)

For instance, the first square side is 1x, while the next square is 1.4x. The first square is rotated at 22.5º thrice to obtain the second square, forming a *marmas* shape (a 16 point-star). If subsequent squares are drawn through these intersection points, the sides we get are 1.1x and 1.2x (Gandotra, 2011) (Fig. 17).

Square-circle interaction in architecture

Gandotra (2011) applied the Square-Circle Sequence (SCS) geometry when reviewing the plan, elevation and roof form of the Hindu temples in India. The researcher used the *Nāgara* style in North India for validating the SCS theory. The cases selected for the study are the square temple of Brahmeshvara in Bhubaneshwar, built in ca. 1060 AD, and the stellate temples of Chennakeshava in Nagalapura, Tumkur District, Karnataka, ca. 1200 AD, and in Jagdamba Devi, Kokamthan, Maharashtra, ca. 14th century AD. In turn, the cases selected for the analysis of the *rekha*, or the curved profile, included the *Latina* style temples of Gujarat and Madhya Pradesh, dating back to the 10th and 11th century.

Meister (1981–1982) mentioned that "the construction of the outer walls of the stellate temples (from 11th century) depends entirely on a constructional geometry using circles to produce turned squares". The researcher studied the Gargaj Mahādeva Temple in Indor, Madhya Pradesh, constructed in ca. mid-8th century. For study purposes, he created sets of circles to locate the squares and *bhadras* projections. Their constructional geometry is shown in Fig. 18.

Application of the square-circle geometry at the Lakshman Temple in Sirpur

The old city of Sirpur, or *Sirpura*, is located in the Central Province of India, on the east bank of River Māhānadī. Cunningham (1884) reported that there are several temples about half a mile to the north-east of the Gandheswar Temple, including a large one called the Lakshman Temple, with a fairly well-preserved *garbhagṛiha* tower. The inscription found at the temple dates it back to the reign of Siva Gupta, around 475 to 500 AD (Cunningham, 1884). The temple is

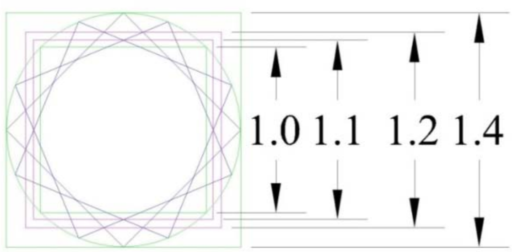

Fig. 17. Ratios (1:1.1:1.2:1.4) between the first square (1x), the second square (1.4x), and the two intermediate squares, generated with the help of the 16-point star shape in the first SCS square (1.1 and 1.2) (drawn by the author)

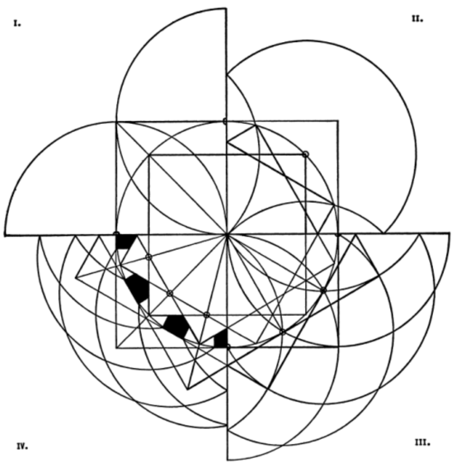

Fig. 18. Sequence showing the circles used for the construction of the outer wall in the Gargaj Mahādeva Temple plan (Meister, 1981–1982).
I: circles at the corners of inscribed and circumscribed squares;
II: circles at the corners of inner turned squares;
III: circles at the corners of outer turned squares;
IV: circles along the width of the *bhadra* projections

dedicated to Lord Vishnu. The sanctum stands on a stone platform: 77 feet long, 39 feet wide, and 7 feet tall. The temple is built up of red brick, with a *garbhagṛiha* and a *maṇḍapa* (which is currently in ruins). The sanctum's area is 22½ square feet outside and 9 square feet 9 square inches inside. It rises 45 feet above the platform.

An attempt was made to apply the geometry of circle-circle intersection (Vesica Piscis) to the proportionate elevation of the Lakshman Temple. The study found that the *Rekha* curvature of the tower, or *Shikhar*, follows the circular arcs of Vesica Piscis. Furthermore, the space between the top of the entablature (*varaṇḍika*) to the bottom of the socle (*vedibandha*) forms a square, which corresponds to the lower part of Vesica Piscis, with a proportion of √2 (Fig. 19).

Result

Now the question arises: what would be the radius of the Vesica Piscis circles used for deriving the *Rekha* curvature of the tower *(Shikhar)*?

To answer it, we apply the geometry of squares and circles (Square-Circle Sequence, or SCS) to the plan of the Lakshman Temple. It starts with a square, which is the inner dimension of the *garbhagṛiha*. Further rotating this square thrice at 22.5º generates the *marma* points (intersection points of the rotating squares). The first square and the circumscribed circle intersect at the *antaral*. The radius of the *fourth* circumscribed circle is used as the radius of circles for Vesica Piscis.

In addition, the dimensions of the square used in the lower part of the temple façade equal the *third* square, whose *marma* points intersect with the outer peripheral wall of the *garbhagṛiha* (Fig. 20). Thus, the *Rekha* curvature of the tower *(Shikhar)* is derived from the radius of the *fourth* circumscribed circle.

Conclusion

Vesica Piscis is the geometry of circle-circle intersection, when two circles of the same radius are drawn with the center of one lying on the circumference of the other. Mostly, this geometry was applied in research when examining building elevation or façade. However, the origin of the circles' radius remained unknown.

Gandotra (2011) introduced a type of square-circle intersection geometry titled the Square-Circle Sequence

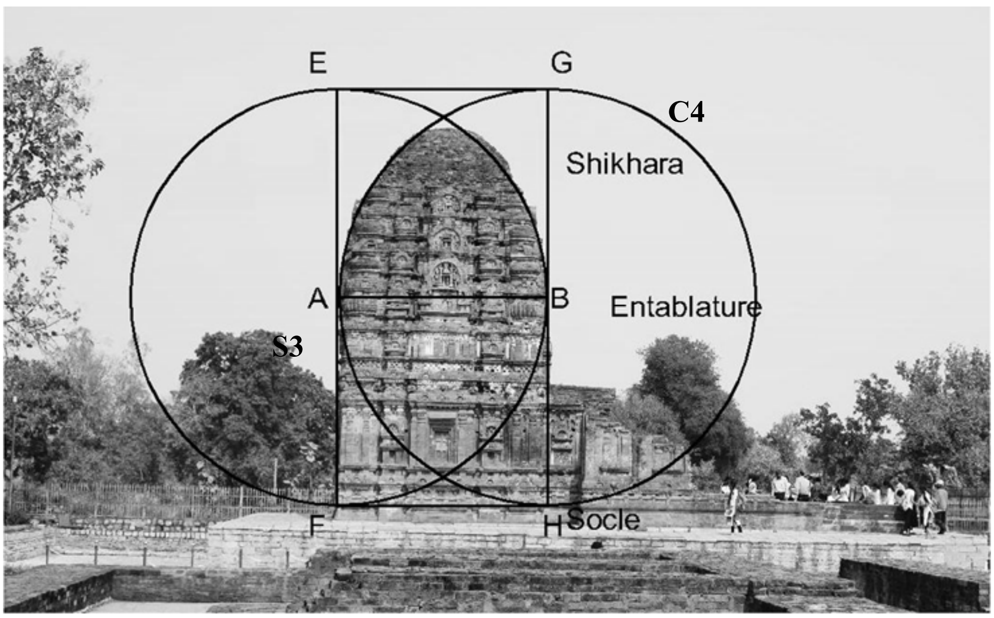

Fig. 19. Geometry of Vesica Piscis applied to the elevation of the Lakshman Temple in Sirpur, Chhattisgarh (photographed and drawn by the author)

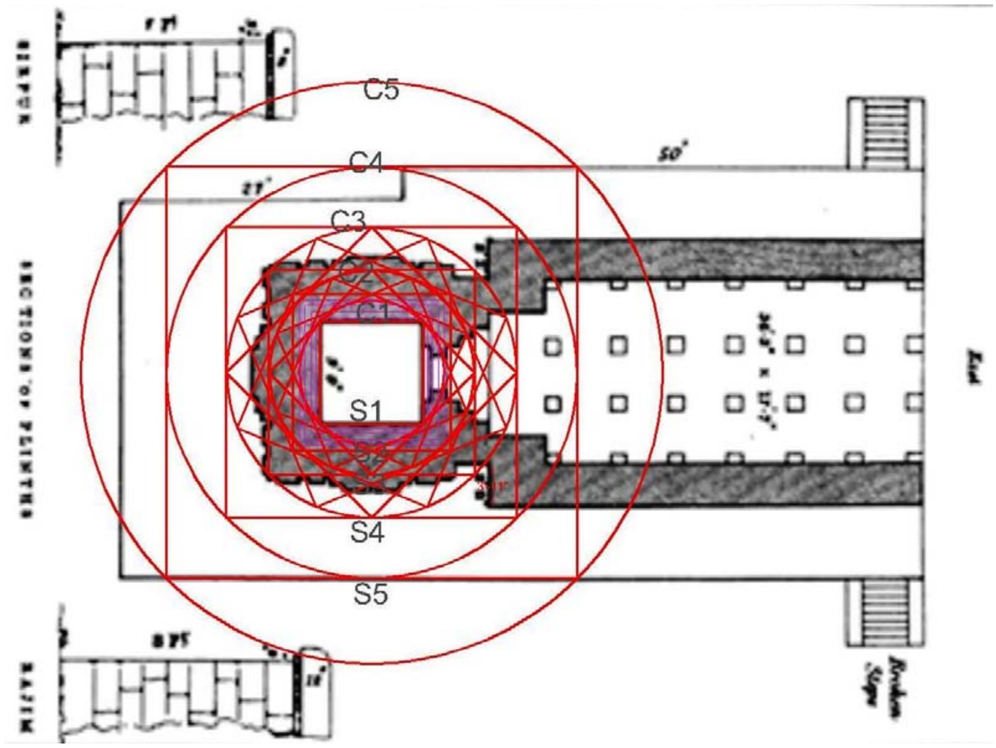

Fig. 20. Applying the Square-Circle Sequence geometry to the plan of the Lakshman Temple in Sirpur, Chhattisgarh (plan sourced from (Cunningham, 1884); the SCS drawn by the author)

(SCS), and applied it to studying the plan and elevation of temples in North India. In her work, the researcher used the circles (obtained from the sequence applied to solving the temple plan geometry) to examine the curvature of the temple's *Shikhara*.

In attempting to derive the constructional geometry behind the elevational form of *Lakshman* Temple in *Sirpur*, we apply the geometries of Vesica Piscis and Square-Circle Sequence. Our study attempts to find a link between these two geometries. Thus, the radius of circles in Vesica Piscis (applied to the curvature of the *Shikhara*) can be obtained by applying the SCS to the temple plan. Also, the square's proportion system and the √2 ratio for Vesica Piscis, as applied to *prāsāda*, can be derived from the SCS in the temple plan.

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ИНТЕРПРЕТАЦИЯ ГЕОМЕТРИИ КВАДРАТОВ И КРУГОВ В АРХИТЕКТУРНОЙ ФОРМЕ ХРАМА ЛАКШМАН В г. СИРПУР (ИНДИЯ)

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Аннотация
Использование базовых фигур, круга и квадрата — один из фундаментальных методов построения конструктивной геометрии любого здания. Круг символизирует жизненную энергию, а квадрат — силу. В мировой истории концепция геометрии восходит к строительным работам в Египте и Вавилоне, где системы пропорций описывались с помощью математических уравнений. Впоследствии они стали известны как теорема Пифагора. В Древней Индии концепция геометрии берет начало от возведения алтарей для ведических жертвоприношений в соответствии с учениями Шульба-сутр. Процесс включал в себя создание кругов и квадратов, преобразование квадратов в круги и наоборот, а его результатом становились алтари различных форм и пропорций. Пересечение основных фигур, квадрата и круга — ключ к конструктивной геометрии зданий. Так, Vesica Piscis, или «рыбий пузырь» — это геометрический элемент, получаемый при пересечении кругов. Исследователи опираются на «рыбий пузырь» при изучении геометрии как древних, так и современных зданий. Аналогичным образом, последовательность «квадрат-круг» (Square-Circle Sequence, SCS) — это метод, где за основу берется пересечение кругов и квадратов. С помощью данного метода Gandotra (2011) исследует конструктивную геометрию индуистских святилищ на севере Индии (нагарских храмов). В работе Meister (1981) пересечение квадрата и круга также используется как конструктивно-геометрический метод для определения системы пропорций индуистских храмов в Индии. Наконец, в настоящем исследовании предпринимается попытка соотнести данные типы конструктивной геометрии с храмом Лакшман в индийском городе Сирпур. В ходе исследования установлено, что конструктивная геометрия здания основана на базовых фигурах: круге и квадрате.

Ключевые слова: геометрия, Шульба-сутры, Vesica Piscis, последовательность «квадрат-круг» (SCS), пересечение кругов, пересечение круга и квадрата, храм Лакшман в Сирпуре.