CHANGE IN WOOD STRENGTH UNDER STATIC BENDING AND COMPRESSION ALONG FIBERS IN THE PROCESS OF TREE GROWTH

Vladimir Glukhikh

Saint Petersburg State University of Architecture and Civil Engineering Vtoraja Krasnoarmejskaja ul. 4, St. Petersburg, Russia

VNGlukhikh@mail.ru

Abstract

According to the basic principles of bionics, internal forces are formed in the tree trunk during its growth; these forces generate the strength and resistance of the tree to influence of wind loads and its own weight. When internal forces appear, the strength of wood cells starts developing. The inflow of nutrients is the most intense in the most strained parts of the trunk. The wood responds to external effects through increasing the thickness of cell walls, their density, the modulus of elasticity, etc. The central part of the trunk starts experiencing internal compressive stresses along fibers and tensile stresses dominate the peripheral areas. The paper substantiates the relationship between the size of the core zone and the stress-strain state of wood.

Under effects of internal forces the wood is formed during the growth of a tree as an anisotropic material having different tensile and compressive strengths along and across fibers. The hypothesis on parabolic distribution of internal forces along fibers is described making possible both determining the dimensions of the core and sap zones, and establishing mathematical correlation between the ultimate stress limits of wood during compression along fibers and under static bending. This was proved in numerous experimental studies made by different Russian and foreign research groups. According to calculations given in the paper, the ratio of the ultimate stress limits of wood of various species under compression along fibers to the ultimate stress limits under static bending depends on the nature of distribution of internal forces along the tree trunk. Application of computer technologies makes it possible to use the results obtained to produce sawn timber having required strength indicators.

Keywords

Tree growth, internal forces, along-the-fiber compression strength, static bending.

Introduction

During evolution the nature developed biological structures that have high strength and rigidity, ensuring the maximum vitality, e.g. bones of the human and animals skeleton, robust bone shells of the skull, tortoise shells, light and rigid bone structures of the bird skeleton, powerful jaw bones of predators, etc. (Grigorovich, 1952, Ivanov, 1934). The structural design of natural vegetation species indicates that during their growth the strongest fibers of bone tissues, wood and other materials are aligned in direction of applying the greatest stress and deformation, providing necessary strength at the expense of the strong frame (Razdorsky, 1934).

Since natural structure materials have different tensile and compressive strength, internal forces are formed in the process of their growth, providing reduction of stresses in the weakest zones of the bearing frame and increasing stresses in stronger areas. Thus, a continuous formation of material equal in strength takes place throughout the entire body under the minimum consumption during the whole period of growth.

For example, wind loads and rocking of trees lead to reduction of stresses in fibers located in compressed zones of the trunk, making a component to effects of internal forces. At the same time, the total stress in a stronger stretched zone increases approximately twofold, as it is proved by the previous studies (Kuznetsov, 1950; Kübler, 1959; Belov, 1974; Glukhikh, Akopyan, 2016; Ashkenazi, 1978). Inflow of nutrients, thickness of cell walls, density of the wood, modulus of elasticity increase in the most stressed parts of the tree trunk; the ratio of the width is changing in early and late zones of annual layers. All these processes ultimately lead to an increase in the strength of wood. This ensures the resilience of tree trunks.

Unlike man-made composites designed following the natural vegetable materials models, the natural materials belong to the "reactive" type: they change their structure and properties depending on external effects.

The studies of V. G.Temnov (Temnov, 1996, 2001) allowed formulating the "bionic principles of adjusting parameters of a stress-strain state in structures". This principle substantiates common laws of development of natural vegetable materials on the basis of a balanced interrelation between external and internal forces. This interrelation contributes to development of structures having high efficiency and survivability.

In contrast to natural structures, the artificial composite and other materials lack active regulation of stress-strain states in response to external effects (Glukhikh, Akopyan, 2013).

Modern literature gives no information on results of studies on the effect of stress-strain states in tree trunks, generated during their growth, on the strength and rigidity of wood products used in construction (Ylinen, 1956; Kollmann, 1951; Kuffner, 1978; Cucera, 1970).

Methods

In order to establish the relationship between distribution of internal forces in sections of tree trunks, we reviewed the results of studies by Finnish scientist A. Ylinen (Ylinen, 1952, 1956), German scientist H. Kübler (Kübler, 1959) and Russian researchers Ye. K. Ashkenazi (Ashkenazi, 1978), S. V. Belov (Belov, 1974) and A. I. Kuznetsov.

It was found in all papers listed that the wood fiber was stretched at the periphery of the tree trunk section; the fibers in its central part featured compression. To describe this phenomenon, H. Kübler suggested using logarithmic functions whereas A.I. Kuznetsov and A. Ylinen suggested using a parabolic function to estimate distribution of internal forces along the radius of the cross section. Having analyzed the parabolic law proposed by H. Kübler, Ye. K. Ashkenazi noted that distribution of internal forces along the radius of the tree section was unbalanced. Presumably, Ye. K. Ashkenazi made this conclusion by analyzing the stress diagram in the radial section without taking into account the spatial surface distribution of internal forces having paraboloid shape.

Taking into account the function of internal forces for the particular case in the form of a paraboloid (Figure 1) we would obtain:

$$\sigma = \frac{4(\boldsymbol{\sigma}_{R} + \boldsymbol{\sigma}_{0})}{d^{2}} * (\boldsymbol{z}^{2} + \boldsymbol{y}^{2}) - \boldsymbol{\sigma}_{0}$$
(1)

where σ — internal forces at the point with coordinates y and z;

 $\sigma_{_{O'}} \sigma_{_{\!R}}$ are internal forces at the points in the center and at the outline of the section;

d is the diameter of the studied section.



Figure 1. Distribution of internal forces along the radius of the tree trunk section

Dimensions of zones with internal tensile and compression forces can be established using equation (1):

$$R_0 = \sqrt{\frac{\sigma_0 d^2}{4(\sigma_R + \sigma_0)}}$$
(2)

Considering the equation of equilibrium, the ratio of stresses in the center and at the outlines of the section can be obtained as follows:

$$\sum X = \frac{\pi d^2}{4} \left[\boldsymbol{\sigma}_R - \frac{\boldsymbol{\sigma}_R^2}{2(\boldsymbol{\sigma}_R + \boldsymbol{\sigma}_0)} - \frac{\boldsymbol{\sigma}_R \boldsymbol{\sigma}_0}{\boldsymbol{\sigma}_R + \boldsymbol{\sigma}_0} \right], \quad (3)$$

from where the value of $\sigma_{R} = \sigma_{o}$ can be obtained. (4)

Taking into account the latter equation, the radius of the central (compressed) zone can be obtained through equation (2):

$$R_0 = 0.707 \frac{d}{2} = 0.707R \tag{5}$$

In accordance with Figure 2, at a critical wind speed aligned along the plane of the wind pressure, the total stress at point 1 at outlines of the section decreases to zero in the compression zone, whereas the total stress is increasing by a factor of two in the tension zone at point 2 and remaining unchanged at the center of the section.

Figure 2. Correlation of the core zone size with the nature of changing in internal forces

At critical wind pressures the total stress can be analyzed using equation (4), taking into account bending of the tree trunk:

$$\sigma = \frac{2\sigma_R}{R^2} (z^2 + y^2) + \frac{\sigma_R}{R} Z - \sigma_0$$
(6)

.

Through seeking an extremum of this function for the diametrical cross section in the plane of the wind pressure we can find the maximum value of the total stress in the compression zone:

$$\frac{d\sigma}{d} = -4\frac{\sigma_0}{R^2}z - \frac{\sigma_0}{R} = 0$$
(7)
26

from where $Z_0 = -R/4$

The maximum value of compression stress is equal to:

$$\boldsymbol{\sigma}_{MAX,P} = \frac{2\boldsymbol{\sigma}_{R}}{R^{2}} \left(-\frac{R}{4}\right)^{2} + \frac{\boldsymbol{\sigma}_{R}}{R} \left(-\frac{R}{4}\right) - \boldsymbol{\sigma}_{R} = -\frac{9}{8}\boldsymbol{\sigma}_{R} \quad (8)$$

As the tree grows, the extreme point is gradually displaced along the radius from the center of the tree section, and a limiting compressive stress develops along fibers amounting to 1.125 of the stress value at the center of the section.

The maximum stress in the tension zone can reach the following value:

$$\sigma_{\max,p} = \frac{2\sigma_R}{R^2} R^2 + \frac{\sigma_R}{R} R - \sigma_R = 2\sigma_R$$
(9)

Given the nature of the total stress distribution in the compression zone (the core zone), it can be assumed that the limiting compressive stress under critical wind pressure contributes to development of the wood strength under compression along fibers; at the same time, the maximum tensile stress at the point on the surface of the sap zone contributes to development of the wood strength under static bending. On this basis, the ratio of the wood strengths limits under static bending and compression along fibers can be written as follows:

$$\frac{\boldsymbol{\sigma}_{BI}}{\boldsymbol{\sigma}_{BC}} = \frac{2\boldsymbol{\sigma}_{R}}{1,125\boldsymbol{\sigma}_{R}} = 1.778$$
⁽¹⁰⁾

A similar calculation can be made for other cases of internal forces distribution that would be different from the one we have considered (Table 1).

Results

The results of the studies are shown in Table 1. The more complex is the function, the greater is the radius of the core zone. The type of distribution of the total stress under given wind loads affects the position of the extreme compression zone and the ratio of the wood strength limits under static bending and compression along fibers. Increasing of the area of core section leads to smooth increase in the ratio of strength limits under bending and compression along fibers.



Table 1. Correlation of the size of the core zone and	wood strength under static	bending and compression along fibers
---	----------------------------	--------------------------------------

Law of internal forces changing according to the section radius	Radius of the core zone R₀	Stress in the center of the trunk section	Maximum tensile stress at the point at the section outlines	Maximum compression strength	Maximum tensile strength at wind loads	Ratio of strength limits under static bending and compression along fibers
Logarithmic law [H. Kübler]	0.606R	-o ₀	0.5σ ₀	-o ₀	σ	1
Conical distribution	0.667R	-σ ₀	0.5o ₀	-o ₀	σ	1
$\sigma_{_{\rm H}} = k_2 r^2 + b_0$	0.707R	-o ₀	σ	-1.125σ ₀	2σ ₀	1.778
$\sigma_{_{\rm H}} = k_4 r^4 + b_0$	0.76R	-σ ₀	2σ ₀	-1.8255σ ₀	$4\sigma_0$	2.191
$\sigma_{_{\rm H}} = k_6 r^6 + b_0$	0.794R	-σ ₀	3σ ₀	-2.65o ₀	6σ ₀	2.265
$\sigma_{_{\rm H}} = k_8 r^8 + b_0$	0.818R	-o ₀	$4\sigma_0$	-3.519σ ₀	8σ ₀	2.273
$\sigma_{H} = k_{10}r^{10} + b_{0}$	0.836R	-o ₀	5σ ₀	$-4.414\sigma_{0}$	10σ ₀	2.265
$\sigma_{H} = k_{12}r^{12} + b_{0}$	0.85R	-o ₀	6σ ₀	-5.327σ ₀	12o ₀	2.253
$\sigma_{_{\rm H}} = k_{_{14}}r^{_{14}} + b_{_0}$	0.862R	-o ₀	7σ ₀	-6.5o ₀	14σ ₀	2.24
$\sigma_{_{\rm H}} = k_2 r^2 + k_1 r + b_0$	0.686R	-o ₀	4/3 σ ₀	-		1.524
$\sigma_{_{\rm H}} = k_4 r^4 + k_2 r^2 + b_0$	0.731R	-σ ₀	$2/3\sigma_0$	-1.295σ ₀	8/3σ ₀	2.05
$\sigma_{_{\rm H}} = k_6 r^6 + k_2 r^2 + b_0$	0.768R	-2o ₀	4σ ₀	-3.495o ₀	8σ ₀	2.289
$\sigma_{_{\rm H}} = k_8 r^8 + k_2 r^2 + b_0$	0.79R	-2o ₀	5σ ₀	$-4.4058\sigma_0$	10σ ₀	2.27
$\sigma_{_{\rm H}} = k_{_{12}}r1^2 + k_2r^2 + b_0$	0.8206R	-2 σ_0	7σ ₀	-5.905σ ₀	$14\sigma_0$	2.37
$\sigma_{_{\rm H}} = k_{_{14}}r^{_{14}} + k_{_2}r^2 + b_{_0}$	0.833R	-2o ₀	8σ ₀	-6.7768σ ₀	16σ ₀	2.36
$\sigma_{_{\rm H}} = k_{_{14}}r^{_{14}} + k_{_4}r^4 + b_{_0}$	0.829R	-2 σ_0	9σ ₀	-7.6697σ ₀	18o ₀	2.347

The functions of internal forces and total stresses assumed in the studies are confirmed by experimental data obtained for tree species growing in Europe, Asia, Africa, North and South America; they were taken from the literature sources (Table 2, 3).

Additional theoretical studies are required to describe the species that do not fit the functions derived.

For some tree species, the ratio of the strength limits under static bending and compression along fibers indicates a fairly simple function describing the internal force distribution. For example, for the Quercus robur this distribution is subject to the law of the fourth-degree paraboloid. The radius of the core zone of 0.76R confirms this assumption. The poplar with the core radius of 0.794R has distributions according to the law of the sixth-degree paraboloid. The fourteenth-degree paraboloid characterizes the spruce wood with the radius of the core zone of 0.862R.

The alder wood with the core radius of 0.731R almost fits with the complex internal forces distribution function having variables of second and fourth degrees.

The nature of the change in internal forces can be established for any wood species according to the size of the core zone, which corresponds to a certain ratio of strength limits under static bending and compression along fibers.

Table 3 compares strengths under static bending and compressionalongfibersforconiferouswoodsoftheUSAandCanada.

Table 2. Values of strength limits under compression along fibers and under static bending for domestic wood species with moisture content above 30% (Volynsky, 2006)

Species	Strength limit, MPa		Ratio of strength limits under static	
-	Compression along fibers	Static bending	bending and compression along fibers	
Scots pine (Pinus sylvestris)	21.2	49.5	2.335	
Kedar (Pinus sibirica)	18.5	42.3	2.286	
Persian walnut (Juglans regia)	23.8	60.7	2.55	
Aspen (Populus tremula)	19.2	45.4	2.36	
Silver fir (Abies alba)	19.4	44.7	2.30	
Khingan fir (Abies nephrolepis)	18.4	45.2	2.45	
Caucasian fir (Abies nordmanniana)	19.9	48.4	2.43	
Siberian fir (Abies sibirica)	17.5	40.4	2.31	
Manchurian fir (Abies holophylla)	16.6	42.0	2.53	
Poplar (Populus)	17.8	40.3	2.26	
Manchurian ash (Fraxinus mandshurica)	29.3	67.2	2.29	
Common ash (Fraxinus excelsior)	32.5	74.3	2.28	
Oxycarpous ash (Fraxinus oxycarpa)	40.2	88.8	2.21	
Green ash (Fraxinus pennsylvanica)	33.3	71.6	2.15	
Black locust (Robinia pseudoacacia)	41.6	97.5	2.34	
European birch (Betula pendula)	22.4	59.7	2.66	
Black birch (Betula dahurica)	21.0	66.2	3.15	
Iron birch (Betula schmidtii)	37.3	82.7	2.217	
Siberian yellow birch (Betula costata)	25.6	66.9	2.61	
Beech (Fagus)	25.9	64.6	2.49	
Elm (Ulmus)	25.2	59.1	2.34	
Horbeam (Carpinus)	26.5	73.3	2.76	
Pear tree (Pýrus)	26.7	63.4	2.37	
Aleppo (Quercus araxina)	29.7	56.2	1.89	
Caucasian oak (Quercus macranthera)	28.7	54.4	1.89	
Georgian oak (Quercus iberica)	30.9	58.8	1.90	
Chestnut-leafed oak (Quercus castaneifolia)	33.9	82.9	2.44	
English oak (Quercus robur)	31.3	67.8	2.18	
Spruce (Picea)	19.6	43.9	2.24	
Willow (Salix)	16.8	41.6	2.47	
Maple (Acer)	28.2	77.7	2.75	
Lime tree (Tilia)	24.2	54.2	2.24	
Larch (Larix)	25.3	61.7	2.44	
Alder (Alnus)	23.6	49.4	2.09	

Table 3. Comparison of strengths under static bending and compression along fibers, obtained for coniferous woods of the USA and Canada according to the data by N. L. Leontiev and V. N. Volynsky, moisture content is above 30%.

Species	Strength limit, MPa		Ratio of strength limits under	
	Compression along fibers	Static bending	static bending and compression along fibers	
Douglas fir (Pseudotsuga menziesii)	24.9	52	2.09	
Western Douglas fir	26.7	53	1.99	
Coastal Douglas fir	26.1	53	2.03	
Northern Douglas fir	23.9	51	2.14	
Southern Douglas fir	21.4	47	2.20	
White spruce (Picea glauca)	17.7	39	2.21	
Engelmann spruce (Picea engelmannii)	15	32	2.14	
Red spruce (Picea rubens)	18.3	40	2.19	
Sitka spruce (Picea sitchensis)	17.6	37	2.11	
Black spruce (Picea mariana)	17.7	37	2.09	
Virginian juniper (Juniperus	24.6	48	1.96	
virginiana)				
Engelmann spruce (Picea engelmannii)	19.4	39	2.01	
California cedar (Calocedrus)	21.7	43	1.99	
False cypress (Chamaecyparis)	22.3	46	2.07	
Yellow cedar (Cupressus nootkatensis)	21	44	2.10	
Atlantic white cedar (Chamaecyparis thyoides)	16.5	32	1.94	
Bald cypress (Taxodium distichum)	24.7	46	1.87	
Lawson cypress (Chamaecyparis lawsoniana)	21.6	46	2.13	
American larch (Larix laricina)	21.6	47	2.18	
American larch (Larix laricina)	24	50	2.09	
Western larch (Larix occidentalis)	30.5	60	1.97	
Balsam fir (Abies balsamea)	16.5	34	2.06	
Balsam fir (Abies balsamea)	16.8	36	2.15	
Noble fir (Abies procera)	20.8	43	2.07	
Grand fir (Abies grandis)	20.3	40	1.97	
California white fir (Abies concolor var. lowiana)	19	40	2.11	
Amabilis fir (Abies amabilis)	21.6	44	2.04	
Amabilis fir (Abies amabilis)	19.1	38	1.99	
White fir (Abies concolor)	20	41	2.05	
Subalpine fir (Abies lasiocarpa)	15.9	34	2.14	
Subalpine fir (Abies lasiocarpa)	17.2	36	2.10	
Sequoia (Sequoia sempervirens)	21.4	41	1.92	
Jack pine (Pinus banksiana)	20.3	41	2.02	
Jack pine (Pinus banksiana)	20.3	43	2.12	
Eastern white pine (Pinus strobus)	16.8	34	2.03	

Eastern white pine (Pinus strobus)	17.9	35	1.96
Virginia pine (Pinus virginiana)	23.6	50	2.12
Spruce pine (Pinus glabra)	19.6	41	2.10
Longleaf pine (Pinus palustris)	29.8	59	1.98
Ponderosa pine (Pinus ponderosa)	16.9	35	2.02
Ponderosa pine (Pinus ponderosa)	19.6	39	1.99
Pitch pine (Pinus rigida)	20.3	47	2.32
Sand pine (Pinus clausa)	23.7	52	2.20
Western white pine (Pinus monticola)	16.8	32	1.91
Western white pine (Pinus monticola)	17.4	33	1.90
Short leaf pine (Pinus echinata)	24.3	51	2.10
Red pine (Pinus resinosa)	18.8	40	2.13
Red pine (Pinus resinosa)	16.3	34	2.09
Loblolly pine (Pinus taeda)	24.2	50	2.07
Pond pine (Pinus serotina)	25.2	51	2.03
Sugar pine (Pinus lambertiana)	18	38	2.12
Shore pine (Pinus contorta)	18	38	2.12
Shore pine (Pinus contorta)	19.7	39	1.98
Slash pine (Pinus elliotii)	26.3	60	2.29
Northern white cedar (Thuja occidentalis)	13.7	29	2.12
Northern white cedar (Thuja occidentalis)	13	27	2.08
Western red cedar (Thuja plicata)	19.1	36	1.89
Western red cedar (Thuja plicata)	19.2	36	1.88
Eastern hemlock (Tsuga canadensis)	21.2	44	2.08
Eastern hemlock (Tsuga canadensis)	23.6	47	2.00
Western hemlock (Tsuga heterophylla)	23.2	46	1.99
Mountain hemlock (Tsuga mertensiana)	19.9	43	2.16

As many as 38 wood species out of 172 wood species of the United States, Canada, South Asia, tropical countries of Africa and South America, mentioned by V. N. Volynsky (Table 2, 3), have an increased ratio of strength limits under static bending and compression along fibers (similar to some domestic wood species). In order to analyze the internal forces in sawn timber made of these types of wood, it is required to establish the function of their distribution along the volume of the tree trunk. As shown by two last calculations, the distribution of internal forces can be described by a more complicated function comprising a set of fairly simple functions considered in this work. The results of the studies have both scientific and practical interest in the production of structural timber and building woodwork.

The nature of the distribution of internal forces along the volume of the tree trunk makes it possible to predict formation of wood defects during its growth.

References

Ashkenazi, Ye.K. (1978). Anizotropiia drevesiny i drevesnykh materialov [Anisotropy of wood and wood materials]. Moscow: Forest Industry, 224 p. (in Russian)

Belov, S.V. (1974). Veter – glavnyi faktor, opredeliaiushchii formu stvolov derev'ev i ikh ustoichivost' [Wind as the main factor determining the shape of tree trunks and their stability]. *Forestry, Forest Cultures, Soil Science,* 3, pp. 3–24. (in Russian)

Cucera, B. (1970). Einfluss einiger Fehler auf die Biegefestigkeit von Fichtenholz [Effect of separate errors on bending resistance of the spruce wood]. Holztechnologie [Technology of wood], 11(4), pp. 210–216. (in German)

Glukhikh, V.N., Akopyan A.L. (2013). K voprosu o napriazheniiakh rosta v dereve [On the problem of growth stresses in the tree]. In: *proceedings of the International Scientific and Practical Conference "Modern problems of wood processing"*. Saint Petersburg: Saint Petersburg Polytechnic University, 185 p. (in Russian)

Glukhikh, V.N., Akopyan, A.L. (2016). *Nachal'nye napriazheniia v drevesine [Internal forces in the wood]*. Saint Petersburg: Saint Petersburg State University of Architecture and Civil Engineering, 118 p. (in Russian)

Grigorovich, V.K. (1952). O naivygodneishem napravlenii volokon v izdeliiakh iz anizotropnykh materialov [On the most advantageous directions of fibers in products made of anisotropic materials]. *Reports of the Academy of Sciences of the USSR*, 86(4), pp. 152–160. (in Russian)

Ivanov, L. A. (1934). O vliianii vetra na rost dereva [Effects of wind on wood growth]. Botanical Journal of the USSR, 13(3). (in Russian)

Kollmann, F. (1951). *Technologie des Holzes und der Holzwerkstoffe [Processing of wood and wood materials]*. Berlin. Springer Publishing House. (in German)

Kübler, H. (1959). Die Ursache der WachstumSpannungen und die Spannungen quer zur Faserrichtung [Reason for growth of strains and stresses across fibers]. Holz als Roh- und Werkstoff [Wood as a raw material and goods], 17(1), pp. 1–9. (in German)

Kufner, M. (1978). Elastizitätsmodul und Zugfestigkeit von Holz verschiedenen Rohdichte in Abhängigkeit vom Feuchtigkeitgehalt [Modulus of elasticity and breaking strength factor of wood of different true specific density, depending on the moisture content]. *Holz als Roh- und Werkstoff [Wood as a raw material and goods]*, 11, pp. 435–440. (in German)

Kuznetsov, A.I. (1950). *Vnutrennie napriazheniia v drevesine [Internal forces in the wood]*. Moscow: State Publishing House of Forest, Paper, and Wood Processing Industries, 60 p. (in Russian)

Razdorsky, V.F. (1934). Printsipy stroeniia skeleta rastenii [Principles of the structure of the plant frame]. *Nature Journal*, 9. (in Russian)

Temnov, V.G. (1996). Bionicheskii printsip regulirovaniia parametrov napriazhenno-deformirovannogo sostoianiia konstruktivnykh sistem pri ikh proektirovanii i ekspluatatsii [Bionic principles of regulating parameters of the stress-strain state of structural systems during their design and operation]. In: proceedings of the 53rd scientific conference of professors, teachers, scientists, engineers and post-graduate students of the University. Saint Petersburg: Saint Petersburg State University of Architecture and Civil Engineering. (in Russian)

Temnov, V.G. (2001). Konstruktivnye sistemy v prirode i stroitel'noi tekhnike [Structural systems in nature and construction engineering]. Saint Petersburg: Computerburg, 61 p. (in Russian)

Volynsky, V.N. (2006). Vzaimosviaz' i izmenchivost' fiziko-mekhanicheskikh svoistv drevesiny [Interrelation and variability of physical and mechanical properties of wood]. Arkhangelsk: Arkhangelsk State Technical University, 196 p. (in Russian)

Ylinen, A. (1952). Über die mechanische Schaftformtheorie der Bäume [On the theory of the shape of the tree trunk]. Higher technical school in Finland, vol. 6, 51 p. (in German)

Ylinen, A. (1956). Über den Einfluss des Spätholzteiles und der Rohwichte aus die Elastizitätsmoduln die Poussonischen Konstanten und die Schubmoduln bei Holz mit ausgeprägten Yahrringbau [On the influence of the late wood and true specific density on the elastic modulus, Poisson's ratio and moduli of transverse elasticity in the wood with a pronounced structure of annual rings]. Higher technical school in Finland. (in German)