# **Civil Engineering**

## ANALYSIS OF BENDING STEEL FIBER REINFORCED CONCRETE ELEMENTS WITH A STRESS-STRAIN MODEL

Aleksey Pavlov, Aleksey Khegay\*, Tatyana Khegay

Saint Petersburg State University of Architecture and Civil Engineering Vtoraja Krasnoarmeyskaya st., 4, Saint Petersburg, Russia

\*Corresponding author: Lhegai@mail.ru

## Abstract

**Introduction**: Fiber concrete is a complex composite material with unique properties. Its behavior is accounted for in steel fiber reinforced concrete elements using simplified stress-strain models. The article describes an approach to the analysis of bending steel fiber reinforced concrete elements using stress-strain diagrams for compression and tension. **Purpose of the study:** The study is aimed to develop a method for the analysis of bending steel fiber reinforced concrete elements using stress-strain diagrams for compression and tension. **Purpose of the study:** The study is aimed to develop a method for the analysis of bending steel fiber reinforced concrete elements using stress-strain diagrams that will be simpler than the existing ones. **Methods**: The study is based on the traditional principles of the analysis of reinforced concrete structures. The method applied is described through equations for the equilibrium of forces affecting the longitudinal axis of the element and the ultimate bending moment. Internal forces in the element cross-section are expressed as relative strains whose diagram is nearly linear in accordance with the Bernoulli's hypothesis. **Results:** We present final equations to determine the load-carrying capacity of a bending element by standard cross-sections. The values of strength and stress-strain properties of fiber reinforced concrete and reinforcement serve as the source data for the calculations.

### **Keywords**

Fiber concrete, steel fiber reinforced concrete, non-linear stress-strain model, fiber reinforced concrete and reinforcement stress-strain diagrams, relative strains.

## Introduction

A trend for the sophistication of design solutions for buildings and structures, especially those made of reinforced concrete, has been observed in modern civil engineering with increasing frequency. Such solutions include space frameworks of buildings with an irregular grid of load-bearing columns and walls that are monolithically bound with ceiling slabs, transition slabs, structurally heterogeneous foundation slabs, frameworks of high-rise buildings with highly loaded massive columns, walls, stiffening cores, foundation slabs, and their connections (Karpenko, 2010). Such structures are characterized by various types of complicated stressstrain behavior. Reinforced concrete may be insufficient for their effective and rational use (Ashour and Wafa, 1993; Evdokimova, 2017).

The risk of progressive fracture determines the need to create structures with increased fracture toughness. In this case, steel fiber introduced into concrete is one of the solutions to the issue. Fiber lays the groundwork for more efficient operation of the structure, which is nowadays especially important due to the growing trend of building unique and high-rise structures.

In view of the above, steel fiber reinforced concrete (SFRC) has become widely used in recent decades. Structures made of SFRC have significant advantages

over reinforced concrete ones, namely: increased crack resistance (fracture toughness), shock resistance, ultimate tensile strength, impermeability, freeze-thaw resistance, reduced shrinkage and creep (Talantova and Mikheyev, 2014; Zhavoronkov, 2017). Fiber reinforcement improves elements' resistance under tension and bending, increases shear strength and burst strength, reduces deformation, as well as reduces the consumption of main reinforcement and makes it possible to give up structural reinforcement (Bakhotskiy, 2013; Valente et al., 2017; Vorontsova, 2017).

Currently, SFRC structures are rarely used in residential construction in Russia. They are still most in-demand in the design of such structures as highway and airfield pavements, long-span bridges, shell-type structures, tunnels, and other kinds of outstanding structures (Brandt, 2008).

The physical nonlinearity of SFRC, without which accuracy and reliability of design solutions decreases, significantly affects the stress-strain state and behavior of all such structures. The main flaw of the existing models and methods of solving physically non-linear reinforced concrete problems is that they reduce the solution to multiple iteration procedures, which, for complex space systems, is an intractable problem even with state-of-the-art computing tools (Karpenko, 2010). Currently, there are various techniques that take into account the specifics of SFRC deformation at load. Thus, in the Regulations SP 360.1325800.2017 (Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation, 2018), analysis using the non-linear stress-strain model (NLSSM) is preferred in the design of buildings and structures since it ensures high reliability of assessment of strength and stress-strain properties (Eryshev, 2018). Nevertheless, standards still allow for analysis using the breaking stress method, which is the simplest task for designers.

Analysis using the NLSSM represents the successive approximations (iterations) method. The method implies breaking down the cross-section of a structural element into elementary sections and calculating average stresses in each of them based on the adopted analytical stressstrain relationships in fiber reinforced concrete (FRC) and reinforcement stress-strain diagrams. The transition from stresses to internal forces is made by means of numerical integration of stresses with respect to the height (width) of the element cross-section. After the condition of the equilibrium of internal forces is checked, stresses in broken-down sections are determined, whose value should not exceed the design values of the strength properties of materials. The problem is solved when the condition of the equilibrium of internal forces is met to the specified accuracy in relation to the previous iteration.

Various types of FRC and reinforcement stress-strain diagrams can be used for analysis: curvilinear (including those with a falling leg), simplified piecewise-linear such as Prandtl diagrams (bilinear, tri-linear, and quad-linear). Since SFRC behavior in compression does not differ much from that of reinforced concrete, stress-strain diagrams for them in the compression zone are taken equivalent. In most cases, during strength and stress-strain analysis, scientists neglect the tension zone in reinforced concrete structures, while it plays a significant role for SFRC. Dispersed reinforcement significantly changes the nature of the stress-strain relationship in the FRC tension zone, which affects the results of tests of FRC beam specimens (Evseyev, 2012; Morozov et al., 2016).

Therefore, since there are many analysis techniques and stress-strain diagrams, there is a particular uncertainty when choosing the stress-strain model for strength and stress-strain analysis to design buildings and structures.

## Problem statement

In practice, the method of analysis using the NLSSM is very time- and labor-consuming and implies the usage of computing tools. Besides, updated standards (Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation, 2018) include no analysis techniques and algorithms that would describe stresses and forces in the element cross-section through FRC strains. It is also necessary to review the analysis results with the use of the existing NLSSM analysis techniques, in terms of breaking stresses and experimental results. That is why we suggest a simpler analysis method for bending steel fiber reinforced concrete elements.

## Analysis basics

• The design model includes the stress diagram for both compression and tension zones of concrete.

• Stress-strain diagrams for FRC and reinforcement are adopted in accordance with the Regulations SP 360.1325800.2017 (Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation, 2018) and SP 63.13330.2018 (Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation, 2019). A tri-linear diagram (Fig. 1) is adopted for the FRC compression zone and a quad-linear diagram (Fig. 2) — for the tension zone. A bilinear diagram (Fig. 3) is adopted for reinforcement with the actual yield strength, and a tri-linear diagram (Fig. 4) — for reinforcement with the conditional yield strength.



Fig. 1. A tri-linear stress-strain diagram for concrete



Fig. 2. A quad-linear stress-strain diagram for FRC



Fig. 3. A bilinear stress-strain diagram for reinforcement



Fig. 4. A tri-linear stress-strain diagram for reinforcement

• Strains along the height of the cross-section are distributed according to the linear law (Bernoulli's hypothesis).

• The analysis is performed relative to the ultimate edge strains of FRC in tension and compression. If the ultimate relative strains of FRC in tension exceed the values of the same parameters for reinforcement, the least value is adopted in the analysis. This is due to the fact that the material is not engaged if the ultimate strains are exceeded.

Since there are no experimental data, the design value of the residual strength of SFRC in tension  $R_{frit}$  is calculated according to equation B.5 or B.7 in the

Regulations SP 360.1325800.2017, and the design value of the residual strength of SFRC in tension  $R_{fbr2}$  is adopted as for subclass *c* (approximately an average value exceeding  $R_{fbr3}$  by 11%) according to Table 2 in the Regulations SP 360.1325800.2017, in line with the assigned class of FRC by the residual strength in tension. The SFRC class by the residual strength in tension is chosen depending on the calculated value of  $R_{fbr3}$  according to Table 2 in the Regulations SP 360.1325800.2017. If the calculated value  $R_{fbr3}$  exceeds the maximum value from the same table,  $R_{fbr3}$  is calculated by equation (1):

$$R_{fbt2} = 1,11R_{fbt3} \tag{1}$$

This is because, for this parameter, there are no analytical relationships that could be associated with the SFRC tensile strength  $R_{fbt}$  , the residual tensile strength,  $R_{ftu3}$  fiber reinforcement coefficient  $\mu_{fv}$  or some other parameters. This value as well as the SFRC subclass in terms of residual strength in tension is calculated experimentally using the method (BS EN 14651:2005+A1:2007 (British Standards Institute, 2005)) based on a test developed by RILEM. According to it, the value of  $R_{fbt2}$  corresponds to the load when the crack mouth opening displacement (CMOD) is 0.5 mm, and the subclass corresponds to the ratio between the values of the guaranteed strength of SFRC in tension when bent R<sub>F2.5</sub> / R<sub>F0.5</sub> (Corporate Standard STO NOSTROY 2.27.125-2013) (National Association of Builders, 2015) (Fig. 5).



Fig. 5. Load vs. CMOD diagram for SFRC

#### Calculation method

Fig. 6 provides a design model of a bending SFRC element.



Fig. 6. Design model of a bending SFRC element

A strain diagram in the element cross-section is constructed on the basis of linear distribution. The strain values are adopted in accordance with the selected stress-strain diagrams for a short-term load:

$$\varepsilon_{fb2} = 0,0035 + 5 \cdot \left(\frac{R_{fb}}{R_b} - 1\right) \cdot 10^{-3} \qquad \text{(at the extreme)}$$

fiber of concrete in the compression zone in accordance with (Rak, 2011);

$$\varepsilon_{fb0} = 0,002$$

 $\varepsilon_{fb1} = \sigma_{fb1} / E_{fb}$  — within the compression zone of FRC;  $\varepsilon_{fb10} = R_{fb1} / E_{fb}$ .

$$\varepsilon_{fbt1} = \varepsilon_{fbt0} + 0,0001$$

 $\mathcal{E}_{fbt2} = 0,004$  — within the tension zone of FRC;

 $\varepsilon_n = \varepsilon_{fbt3} = 0,02 - 0,0125 (R_{fbt3} / R_{fbt2} - 0,5)$  — at the extreme fiber of concrete in the tension zone at

 $\varepsilon_{fbt3} \le \varepsilon_{s2}$ ,  $\varepsilon_{s2}$  — at  $\varepsilon_{fbt3} > \varepsilon_{s2}$ ;  $\varepsilon'_{sj}$  — at the level of the center of gravity of compressive reinforcement:

of the center of gravity of compressive reinforcement;

 $\mathcal{E}_{si}$  — at the level of the center of gravity of tensile reinforcement.

The stress diagram in the compression zone includes three sections with a height of  $h_1$ ,  $h_2$  and  $h_3$ , and in the tension zone — four sections with a height of  $h_4$ ,  $h_5$ ,  $h_6$ and  $h_7$ . Taking into account the forces in compressive and tensile reinforcement, we derive the equation for the equilibrium of forces affecting the longitudinal axis of the element:

$$N_{fb1} + N_{fb2} + N_{fb3} + N'_{sj} - N_{fbt1} - N_{fbt2} - N_{fbt3} - N_{fbt4} - N_{si} = 0.$$
(2)

Forces in FRC are determined by the areas of corresponding sections of the stress diagrams, and in reinforcement — by the stresses and the area of the cross-section.

$$N_{fb1} = R_{fb}bh_{1}, N_{fb2} = 0,8R_{fb}bh_{2},$$

$$N_{fb3} = 0,3R_{fb}bh_{3}, N_{fb1} = 0,5R_{fbt}bh_{4},$$

$$N_{fbt2} = R_{fbt}bh_{5}, N_{fbt3} = 0,5(R_{fbt} + R_{fbt2})bh_{6},$$

$$N_{fbt4} = 0,5(R_{fbt2} + R_{fbt3})bh_{7},$$

$$N'_{si} = \sigma'_{si}A'_{si}, N_{si} = \sigma_{si}A_{si}.$$
(3)

The height of the compression zone is calculated through the ratio  $\varepsilon_{fb2} / x = \varepsilon_n / (h - x)$ :

$$x = \frac{\varepsilon_{fb2}h}{\varepsilon_{fb2} + \varepsilon_n} = \frac{\varepsilon_{fb2}}{\chi},$$
(4)

where  $\chi$  is the curvature of the element calculated by the following equation:

$$\chi = \frac{1}{\rho} = \frac{\varepsilon_{fb2} + \varepsilon_n}{h},\tag{5}$$

where  $\rho$  is the element curvature radius; h is the element height.

The section height values corresponding to the stressstrain diagram of SFRC are calculated on the basis of ratios between the strains, the height of the compression zone and the height of the section. The result is as follows:

$$h_{1} = \frac{\varepsilon_{fb1}}{\chi}, h_{2} = \frac{\varepsilon_{fb0} - \varepsilon_{fb1}}{\chi}, h_{3} = \frac{\varepsilon_{fb1} - \varepsilon_{fb2}}{\chi},$$
$$h_{4} = \frac{\varepsilon_{fb10}}{\chi}, h_{5} = \frac{\varepsilon_{fb11} - \varepsilon_{fb10}}{\chi},$$
$$h_{6} = \frac{\varepsilon_{fb12} - \varepsilon_{fb11}}{\chi}, h_{7} = \frac{\varepsilon_{n} - \varepsilon_{fb12}}{\chi}.$$
(6)

The strains in compressive and tensile reinforcement are calculated using the following ratios:

$$\frac{\varepsilon_{fb2}}{x} = \frac{\varepsilon'_{sj}}{x - a'_{j}}, \quad \frac{\varepsilon_{fb2}}{x} = \frac{\varepsilon_{si}}{h - x - a_{i}}, \quad (7)$$

where  $a'_{j}$  and  $a_{i}$  are the distances from the center of gravity of compressive and tensile reinforcement to the extreme fiber of FRC in the compression and tension zones, respectively.

The result is as follows:

$$\varepsilon'_{sj} = \varepsilon_{fb2} - \chi a'_j, \ \varepsilon_{si} = h\chi - \varepsilon_{fb2} - a_i\chi.$$
(8)

The ultimate relative strains of reinforcement are limited by:  $\varepsilon_{s2} = 0,025$  — for reinforcement with the actual yield strength;  $\varepsilon_{s2} = 0,015$  — for reinforcement with the conditional yield strength.

Taking into account equations (3) and (6), equation (2) will take the following form:

$$\frac{R_{fb}b}{\chi} \left( \varepsilon_{fb2} - 0, 2\varepsilon_{fb0} - 0, 5\varepsilon_{fb1} \right) + \sigma'_{sj} A'_{sj} - \frac{b}{2\chi} \left( R_{fbt} \left( \varepsilon_{fbt1} - \varepsilon_{fbt0} + \varepsilon_{fbt2} \right) + R_{fbt2} \left( \varepsilon_n - \varepsilon_{fbt1} \right) + R_{fbt3} \left( \varepsilon_n - \varepsilon_{fbt2} \right) \right) - \sigma_{si} A_{si} = 0, \quad (9)$$

where strains in reinforcement are calculated depending on the adopted stress-strain diagram. It should be noted that with strains  $\mathcal{E}_n \leq \mathcal{E}_{fbt2}$ , there will be no section with a height of  $h_7$ , and the diagram in the tension zone will be tri-linear since the fourth section of the tensile diagram starts at strains exceeding  $\mathcal{E}_{fbt2}$ . Therefore, the member  $R_{fbt3}\left(\mathcal{E}_n - \mathcal{E}_{fbt2}\right)$  in equation (9) will not be considered.

Thus, we obtain the equation for the equilibrium of forces, expressed through strains. The first two members that determine the forces in concrete and reinforcement of the compression zone are adopted to be positive, the second ones that determine the forces in concrete and reinforcement of the tension zone are adopted to be negative. The condition of the equilibrium will be met if these two parts are equal. Therefore, we need to determine edge strains in the element cross-section when equation (9) is satisfied. Two options are possible in the calculation of the sum of the forces:

1. Option 1 — the sum of the members in the left part is less than zero.

2. Option 2 — the sum is more than zero.

In the first case, we need to reduce strains  $\varepsilon_n$  at the constant value of strain  $\varepsilon_{fb2}$  (thereby increasing the height of the concrete cmpression zone) to a certain value when condition (9) is met with sufficient accuracy. This condition can be written as follows:

$$\left|\sum_{i=1}^{n} N_{i}\right| \le 0,1 \text{ (kH)}.$$
(10)

Strains  $\mathcal{E}_n$  are calculated by equation (11):

$$\varepsilon_{n} = \frac{2R_{fb} \left(\varepsilon_{fb2} - 0, 2\varepsilon_{fb0} - 0, 5\varepsilon_{fb1}\right) - R_{fbt2} + R_{fbt3} - R^{A}}{R_{fbt2} + R_{fbt3} - R^{A}}$$

$$+ R^{A} \varepsilon_{fb2} + R_{fbt2} \varepsilon_{fbt1} + R_{fbt3} \varepsilon_{fbt2}, \qquad (11)$$

where the force in compressive and tensile reinforcement is calculated through the yield strength of reinforcement steel taking into account concrete reduction (for  $R_{sc}$ ), by equation (12):

$$R^{A} = \frac{2}{bh} \Big( R_{sc} A'_{sj} - R_{s} A_{si} \Big).$$
(12)

If strains of compressive or tensile reinforcement by equation (8) are less than the values of  $\mathcal{E}_{s0}$ , the second iteration is made. In it, stresses  $\sigma'_{sj} = \mathcal{E}'_{sj}E_s$ ,  $\sigma_{si} = \mathcal{E}_{si}E_s$  are used instead of  $R_{sc}$  and  $R_s$  in equation (12).

With strains  $\varepsilon_n \leq \varepsilon_{fbt2}$  the members  $R_{fbt3}\varepsilon_{fbt2}$  and  $R_{fbt3}$  are not taken into account in equation (11).

In the second case, we need to reduce strains  $\varepsilon_{fb2}$  at the constant value of strains  $\varepsilon_n = \varepsilon_{fb13}$  or  $\varepsilon_n = \varepsilon_{s2}$  (depending on what relative strains are less) to meet condition (9) with the set accuracy (10).

Strains  $\mathcal{E}_{fb2}$  are calculated by equation (13):

$$\varepsilon_{fb2} = \frac{R_{fb} \left( 0, 4\varepsilon_{fb0} + \varepsilon_{fb1} \right) + R_{fbt} \left( \varepsilon_{fbt1} - \varepsilon_{fbt0} + \varepsilon_{fbt2} \right) +}{2R_{fb} + R^{A}}$$
$$\frac{+ R_{fbt2} \left( \varepsilon_{fbt3} - \varepsilon_{fbt1} \right) + R_{fbt3} \left( \varepsilon_{fbt3} - \varepsilon_{fbt2} \right) - R^{A} \varepsilon_{fbt3}}{. (13)}$$

The condition of the strength of bending SFRC elements is written as  $M \leq M_{ult}$ , where M is the moment from external forces;  $M_{ult}$  is the ultimate bending moment that the element cross-section can take. The value of  $M_{ult}$  is determined relative to the neutral axis. The distances from the forces occurring in concrete and reinforcement to this axis will be as follows:

$$z_1 = \frac{\varepsilon_{fb2} + \varepsilon_{fb0}}{2\chi}, \ z_2 = \frac{13\varepsilon_{fb0} + 11\varepsilon_{fb1}}{24\chi}$$

$$z_{3} = \frac{2\varepsilon_{fb1}}{3\chi}, z_{4} = \frac{2\varepsilon_{fbt0}}{3\chi}, z_{5} = \frac{\varepsilon_{fbt0} + \varepsilon_{fbt1}}{2\chi},$$

$$z_{6} = \frac{R_{fbt} \left(2\varepsilon_{fbt1} + \varepsilon_{fbt2}\right) + R_{fbt2} \left(\varepsilon_{fbt1} + 2\varepsilon_{fbt2}\right)}{3\chi \left(R_{fbt} + R_{fbt2}\right)},$$

$$z_{7} = \frac{R_{fbt2} \left(2\varepsilon_{fbt2} + \varepsilon_{n}\right) + R_{fbt3} \left(\varepsilon_{fbt2} + 2\varepsilon_{n}\right)}{3\chi \left(R_{fbt2} + R_{fbt3}\right)},$$

$$z_{si} = \frac{h\chi - \varepsilon_{fb2} - a_i\chi}{\chi}, \ z'_{sj} = \frac{\varepsilon_{fb2} - a_j\chi}{\chi}.$$
 (14)

The equation for the ultimate bending moment with account for dependences (3), (6) and (14) will take the following form:

$$M_{ult} = \frac{R_{fb}b}{24\chi^2} \left( 12\varepsilon_{fb2}^2 - 1, 6\varepsilon_{fb0}^2 - 4\varepsilon_{fb1}^2 - 1, 6\varepsilon_{fb0}\varepsilon_{fb1} \right) + \frac{b}{6\chi^2} \left[ R_{fbt} \left( 3\varepsilon_{fb1}^2 - \varepsilon_{fb0}^2 \right) + R^2 + R^3 \right] + \sigma_{si}A_{si} \left( \frac{h\chi - \varepsilon_{fb2} - a_i\chi}{\chi} \right) + \sigma_{sj}'A_{sj}' \left( \frac{\varepsilon_{fb2} - a_j'\chi}{\chi} \right), \quad (15)$$

where the moments from the tension zone of concrete with a height of  $h_6$  and  $h_7$  equal to the following:

$$R^{2} = R_{fbt} \left( \varepsilon_{fbt2}^{2} - 2\varepsilon_{fbt1}^{2} + \varepsilon_{fbt1} \varepsilon_{fbt2} \right) + R_{fbt2} \left( 2\varepsilon_{fbt2}^{2} - \varepsilon_{fbt1}^{2} - \varepsilon_{fbt1} \varepsilon_{fbt2} \right),$$
(16)

$$R^{3} = R_{fbt2} \left( \varepsilon_{n}^{2} - 2\varepsilon_{fbt2}^{2} + \varepsilon_{fbt2} \varepsilon_{n} \right) + R_{fbt3} \left( 2\varepsilon_{n}^{2} - \varepsilon_{fbt2}^{2} - \varepsilon_{fbt2} \varepsilon_{n} \right).$$
(17)

It should also be noted that with strains  $\mathcal{E}_n \leq \mathcal{E}_{fbt2}$ , there will be no section with a height of  $h_{\gamma}$ , and the member (17) will not be taken into account in equation (15).

## Conclusions

The growing popularity of SFRC structures requires special attention of designers to strength and stress-strain analysis. In this regard, analysis using the non-linear stress-strain model with the use of various stress-strain diagrams for materials is highly important for studies. Firstly, such analysis is very time- and labor-consuming; secondly, it is used as verification analysis, which is not very convenient in terms of design.

To eliminate these flaws, a method based on elastoplastic stress-strain diagrams for FRC and reinforcement has been developed. In this method, the value of the element curvature is adopted as a variable of successive approximation, and the forces in the FRC compression and tension zones to the neutral axis are found as the areas of the diagram sections with their own centers of gravity.

This method makes it possible to avoid numerical integration with multiple iterations along the entire height

of the cross-section, since only one value is a variable. Besides, it allows us to calculate the load-bearing capacity of SFRC elements, which, in its turn, depends on the strength and stress-strain properties of base materials.

However, this method has a flaw: analysis uses such characteristics of FRC that can be determined only by means of experiments.

## Acknowledgments

The authors express their gratitude to Vladimir Mirovich Popov, PhD in Engineering, Associate Professor at the Department of Reinforced Concrete and Masonry Constructions of the Saint Petersburg State University of Architecture and Civil Engineering, for his advice and peer review.

## References

Ashour, S. A. and Wafa, F. F. (1993). Flexural behavior of high-strength fiber reinforced concrete beams. *ACI Structural Journal*, 90 (3), pp. 279–287.

Bakhotskiy, I. V. (2013). Experimental and theoretical investigations of stress-strain behavior of fiber-reinforced concrete members exposed to bending with torsion. *Bulletin of Civil Engineers*, 4, pp. 48–53.

Brandt, A. M. (2008). Fibre reinforced cement-based (FRC) composites after over 40 years of development in building and civil engineering. *Composite Structures*, 86 (1–3), pp. 3–9. DOI: 10.1016/J.COMPSTRUCT.2008.03.006.

British Standards Institute (2005). BS EN 14651:2005+A1:2007. Test method for metallic fibre concrete – Measuring the flexural tensile strength (limit of proportionality (LOP), residual), 20 p.

Eryshev, V. A. (2018). Deformation method for calculating the strength of reinforced concrete bending elements using deforming diagrams for elastoplastic materials. *Systems. Methods. Technologies*, 1, pp. 79–84. DOI: 10.18324/2077-5415-2018-1-79-84.

Evdokimova, T. S. (2017). Stress-strain behavior and strength analysis of fiber-reinforced concrete elements subjected to skew bending. PhD Thesis in Engineering. Saint Petersburg: Saint Petersburg State University of Architecture and Civil Engineering.

Evseyev, E. N. (2012). Testing of fiber according to European standards. Concrete Technologies, 5–6, pp. 31–32.

Karpenko, S. N. (2010). *Reinforced concrete stress-strain models in increments and structural analysis methods.* DSc Thesis in Engineering. Moscow: Research Institute of Construction Physics of the Russian Academy of Architecture and Construction Sciences.

Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation (2018). Regulations SP 360.1325800.2017. Steel fiber reinforced concrete structures. Design rules. Moscow: Standartinform, 70 p.

Ministry of Construction Industry, Housing and Utilities Sector of the Russian Federation (2019). Regulations SP 63.13330.2018. Concrete and reinforced concrete structures. General provisions. Revised edition of SNiP 52-01-2003. Moscow: Standartinform, 143 p.

Morozov, V., Khegai, A., Evdokimova, T. and Opbul, E. (2016). Experimental diagrams of fiber concrete straining under tension and compression and their implementation in calculation of bearing capacity of fiber-reinforced concrete flexural elements. *Materials Science Forum*, 871, pp. 160–165. DOI: 10.4028/www.scientific.net/MSF.871.160.

National Association of Builders (2015). Corporate Standard STO NOSTROY 2.27.125-2013. Underground space development. Structures of transport tunnels made of fiber-reinforced concrete. Design and construction rules. Moscow: Ordzhonikidze Central Institute of Standard Design, 104 p.

Rak, N. A. (2011). A method of structural analysis for steel fiber reinforced concrete structures using steel fiber reinforced concrete and bar reinforcement stress-strain diagrams. In: *Proceedings of the 3<sup>rd</sup> International Symposium "Issues of Modern Concrete and Reinforced Concrete"*. Minsk.

Talantova, K. V. and Mikheyev, N. M. (2014). *Steel fiber concrete and constructions based thereon.* Saint Petersburg: Emperor Alexander I St. Petersburg State Transport University, 275 p.

Valente, T., Gonçalves, D., Lourenço, L., Costa, I. and Barros, J. (2017). Fiber reinforced concrete. *Concrete Goods and Constructions*, pp. 54–62.

Vorontsova, N. S. (2017). Experimental investigations of fiber reinforced concrete members subjected to biaxial bending. *Bulletin of Civil Engineers*, 4, pp. 66–71. DOI: 10.23968/1999-5571-2017-14-4-66-71.

Zhavoronkov, M. I. (2017). *Developing methods for the determination of fiber-reinforced concrete crack resistance characteristics*. PhD Thesis in Engineering. Saint Petersburg: Saint Petersburg State University of Architecture and Civil Engineering.

## РАСЧЕТ ИЗГИБАЕМЫХ СТАЛЕФИБРОЖЕЛЕЗОБЕТОННЫХ ЭЛЕМЕНТОВ С УЧЕТОМ ДЕФОРМАЦИОННОЙ МОДЕЛИ

Алексей Сергеевич Павлов, Алексей Олегович Хегай\*, Татьяна Сергеевна Хегай

Санкт-Петербургский государственный архитектурно-строительный университет 2-ая Красноармейская ул., 4, Санкт-Петербург, Россия

\*E-mail: Lhegai@mail.ru

## Аннотация

Фибробетон сложный композитный материал с уникальными свойствами. Его работа учитывается в сталефиброжелезобетонных элементах по упрощенным моделям деформирования. В статье рассматривается подход к расчету сталефиброжелезобетонных изгибаемых элементов с использованием диаграмм деформирования фибробетона на сжатие и растяжение. Цель исследования: Разработка метода расчета сталефиброжелезобетонного изгибаемого элемента по диаграммам деформирования, отличающегося более простым подходом по сравнению с существующими. Методы: В основе метода лежат классические принципы расчета железобетонных конструкций. Метод описывается уравнениями равновесия усилий на продольную ось элемента и предельным изгибающим моментом. Внутренние усилия в сечении элемента выражаются через относительные деформации, эпюра которых имеет линейное очертание в соответствие с гипотезой плоских сечений. Результаты: Представлены итоговые расчетные формулы для определения несущей способности изгибаемого элемента по нормальным сечениям. Исходными данными для расчета являются значения прочностных и деформативных характеристик фибробетона и арматуры.

## Ключевые слова

Фибробетон, сталефиброжелезобетон, нелинейная деформационная модель, диаграммы деформирования фибробетона и арматуры, относительные деформации.