

## ARCHITECTURAL DESIGNS INSPIRED BY NATURE AND MATHEMATICAL MODELS

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### Abstract

**Introduction:** This paper explores how nature-inspired mathematical models and principles are applied in architectural design and how these approaches contribute to innovative and sustainable solutions. Architectural structures incorporating nature-inspired and mathematical models have been widely studied in the literature. However, research examining the relationship between nature and mathematics specifically within the field of architecture remains limited. Addressing this gap, the study investigates the interplay between mathematics and nature in architectural design. Its contribution lies in providing a holistic examination of mathematical models in architecture. Within the scope of the study, five examples demonstrating different mathematical theorems were analyzed. **Methods:** The structures were evaluated using qualitative content analysis, while the mathematical models were identified through visual content analysis. The findings are summarized in a table. **Conclusion:** Architecture is profoundly shaped by the relationship between nature and mathematics. Analyzing the mathematics inherent in nature and applying these principles in design serves as a guide for creating aesthetically pleasing, sustainable, and innovative buildings.

**Keywords:** mathematical model, nature-inspired architecture, golden ratio, fractal geometry, mathematics in architecture.

### Introduction

Humans are inspired by and learn from nature in all their creations. However, this influence is not unidirectional, as humans also attribute order to nature (Peker, 2017). The search for order within the apparent chaos of nature reflects human efforts to understand and reconstruct the physical world. In this process, mathematics serves as a tool to interpret the complexity and patterns of nature. Ancient philosophers argued that the fundamental structure of the universe is rooted in mathematics and that this serves as the creative element of nature (Cooper, 1997). This perspective has influenced architecture since antiquity. Architecture materializes humanity's relationship with nature by embodying mathematical order and harmony in the physical environment. From prehistoric times through Ancient Greece, Rome, the Renaissance, and Modern Architecture, mathematical principles derived from nature have shaped architectural design.

The relationship between architecture and mathematics is evident in the direct application of fundamental mathematical concepts — such as proportion, symmetry, and geometry — to building design. Architects employ mathematics to achieve structures that are balanced aesthetically and functionally. Throughout history, mathematical

principles have provided stability and harmony in architectural forms. Furthermore, the mathematics of nature enables the imitation of natural forms and the integration of buildings with their surroundings. In this way, the mathematics of nature has long served as a source of inspiration for both aesthetic and functional aspects of architecture (Aejaz and Yasmeen, 2023; Kavurmacioğlu and Arıdağ, 2013).

The use of mathematics and geometry in architecture is not only a compositional tool but also a language of representation, employed throughout history to convey cultural, spiritual, and even cosmic meanings in architectural works. As Salvadori (1968) emphasizes, although architecture is grounded in the physical world, its connection with abstract mathematical principles is indispensable. No architectural work can exist without mathematical foundations such as measurements, proportions, and geometry. For instance, the use of geometry to express cosmic themes in ancient Greek and Roman architecture illustrates how the mathematical order inherent in nature influences architectural design (Ostwald and Williams, 2015). The direct relationship between the mathematical order found in nature and its application in architecture demonstrates how mathematics has shaped both the visual and structural language of buildings across cultures

and historical periods. During the Renaissance, architects, artists, and designers extensively explored the relationship between mathematics and nature. They documented the golden section ratios and applied them in remarkable works, including sculptures, paintings, and architectural designs (Akhtaruzzaman and Shafie, 2011).

Mathematical models represent the structure and organization of forms found in nature and have been employed in architectural design for centuries (Nowak, 2015). The golden ratio, one of nature's mathematical properties, can be observed across a wide range of length scales, from the galactic to the atomic (Marples and Williams, 2022). Using Fibonacci numbers, the golden ratio forms a golden spiral that appears ubiquitously in nature, such as in shells, pine cones, the arrangement of seeds in a sunflower head, and even in galaxies. Adolf Zeising, a mathematician and philosopher, concluded that the golden ratio operates as a universal law when studying natural phenomena (Akhtaruzzaman and Shafie, 2011). Fractal geometry, in turn, explains many natural formations, from the branching structures of plants to the rings of Saturn, reflecting the self-similarity inherent in nature. Examples of fractal structures include blood vessels, lungs, river networks, lightning, tree branches, rocky coastlines, and weather patterns (Bovill, 1996; Hacısalıhoğlu, 2015). The concept of tessellation, important in aesthetics, mathematics, chemistry, and molecular science, refers to the covering of a plane without overlaps or gaps. Various tessellation patterns can be observed in self-organizing systems in biology and nanotechnology (Cheng et al., 2018). Similarly, the Voronoi diagram describes a self-organizing system of biological structures, as seen in dragonfly wings, turtle shells, honeycombs, and sea urchin shells (Nowak, 2015). Natural examples such as tree branches, rocks, and Romanesco broccoli also exhibit Voronoi structures (Kornev, 2018; Zhao et al., 2016). Modern design increasingly relies on mathematical models to understand the principles of self-organization in biological structures. The use of computational geometry in architecture and urban planning offers new opportunities for designing structures and elements that integrate these natural processes and patterns (Nowak, 2015).

To summarize the information presented so far, the relationship between architecture and mathematics is grounded in the use of proportion, symmetry, geometry, and mathematical models in building design. Concepts such as the golden ratio, fractal geometry, and tessellation have been widely employed in architecture to achieve both aesthetic and structural balance. Throughout history, the mathematics of nature has inspired the imitation of natural forms in architecture and facilitated the harmonious integration of buildings with their

surroundings. Since the Renaissance, mathematical proportions have gained particular importance for ensuring both structural stability and visual appeal. Fractal and geometric forms observed in nature are now reinterpreted and applied in architecture through computational design techniques. From this perspective, the present study examines the use of mathematical models — including the golden ratio, fractal geometry, tessellation, Voronoi diagrams, and the Sierpiński Triangle — in architectural design. The relationship between mathematics, nature, and architecture is explored through a selection of building examples. A unique aspect of this study is the presentation of a method for evaluating the interplay between nature and mathematics, highlighting how buildings incorporate mathematical models in facades, floor plans, and three-dimensional forms. For this purpose, abstractions were created based on photographs of buildings obtained from various websites and books. Traces of mathematical models were identified across different dimensions, reflecting patterns inspired by nature. These patterns relate to nature not only at the structural level but also in terms of spatial organization.

#### ***Mathematical Models and Principles Used in Architectural Design***

Mathematical principles are frequently employed in architecture to achieve aesthetic appeal, structural efficiency, and harmony with nature. These principles contribute to the modeling of natural systems and the design of architectural structures. Mathematical models have been applied in architecture in various forms since antiquity. Among these, the golden ratio is the most widely recognized and utilized. The golden ratio, or golden proportion, represents an irrational number approximately equal to 1.6180339887 and is denoted by the Greek letter phi ( $\phi$ ). Due to its unique and intriguing properties, the golden ratio — also referred to as the golden section — has been extensively studied by researchers and mathematicians (Akhtaruzzaman and Shafie, 2011). It generally describes the mathematical proportion between sizes and shapes (Fig. 1). Valued for its

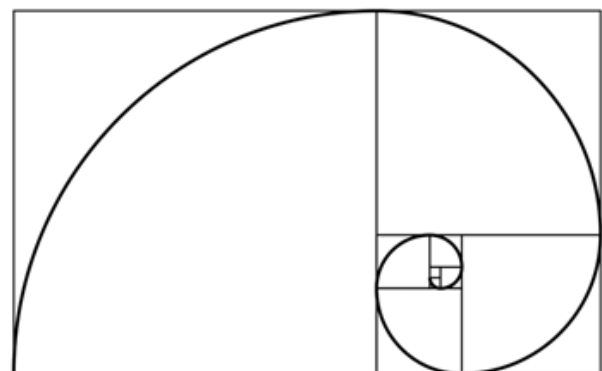


Fig. 1. Golden ratio (drawn by the authors)

aesthetic appeal, it is believed to impart beauty and harmony in nature and art (Bergil, 1993). According to the architectural theorist Neufert, the golden ratio functions as an architectural ratio and is considered an inherent law in architecture. Le Corbusier, meanwhile, viewed it as a natural rhythm reflected in the human organism. The golden ratio is applied across various art forms to create visual balance and harmony. In architecture, it has been used in both floor plans and facades. Prominent historical examples include the Pyramid of Cheops and the Parthenon, which exemplify the application of golden ratio principles (Peker, 2017).

Fractal geometry represents a form of complex, repetitive, and continuous patterns (Fig. 2) (Bovill, 1996). It is used to describe structures that cannot be defined by Euclidean geometry. The forms of natural objects rarely correspond to the rigid shapes of Euclidean geometry, which is insufficient for explaining the complexity of natural structures. Consequently, the geometry of nature is best described by fractal geometry (Mandelbrot, 1982). A defining characteristic of fractal structures is their self-similarity across different scales. Regardless of scale, the degree of irregularity in a fractal remains consistent, producing ordered complexity within apparent randomness. This property allows fractals to be used effectively to describe and analyze natural forms. In architecture, fractal principles are applied to examine the complexity, repetition, and self-similarity of floor plans and facades (Hagerhall et al., 2004).

Tessellation refers to the complete covering of a plane with repeating geometric shapes (Fig. 3)

(Grünbaum and Shephard, 1987). Tessellations, which have been employed in engineering, art, and architecture since antiquity, can be categorized into three main types: regular, semi-regular, and irregular. Regular tessellations are created from shapes such as squares, rectangles, and triangles, while semi-regular tessellations arise from combining different polygons at a common vertex. Irregular tessellations are formed by arranging groups of shapes in different combinations. In architecture, tessellations are widely applied in exterior claddings and ceramic surfaces, serving both aesthetic and structural purposes (Gazi and Korkmaz, 2015).

Voronoi diagrams, originating in the 17<sup>th</sup> century and first formally illustrated by Georgy Voronoi in 1903, represent another important mathematical model (Fig. 4). As a data segmentation method, Voronoi diagrams are particularly suited for solving closest-point problems (Vassilev and Eades, 2013). Constructed from a set of points generating polygonal cells, this system defines and measures relationships between points in various fields. With advances in computational technologies, Voronoi diagrams have become a valuable tool in architectural design, yielding effective results in facades, spatial organization, urban planning, mapping, and analytical studies (Sack and Urrutia, 1999). Moreover, Voronoi diagrams are widely observed in nature, including in cell division, animal skin patterns, and leaf surfaces (Nowak and Rokicki, 2016).

The Sierpiński triangle, introduced by the Polish mathematician Waclaw Sierpiński in 1915, exemplifies a self-replicating system of progressively

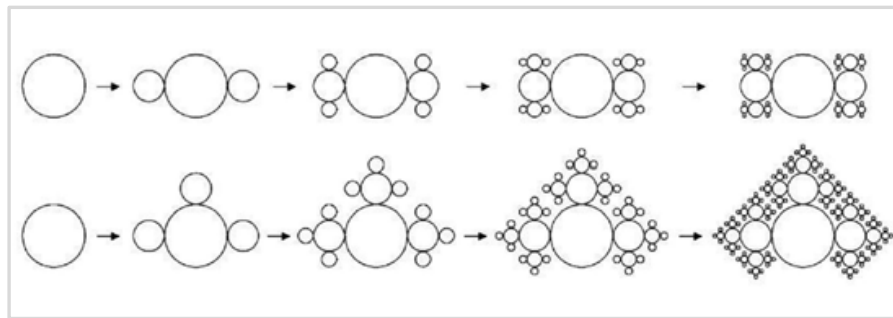


Fig. 2. Example of a fractal process (Rian et al., 2007)

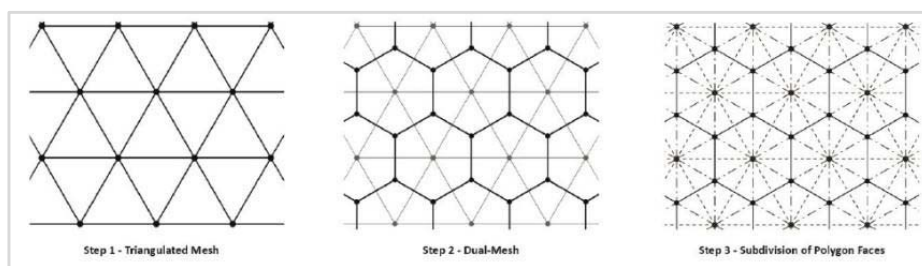


Fig. 3. Tessellation process (Chandra et al., 2015)

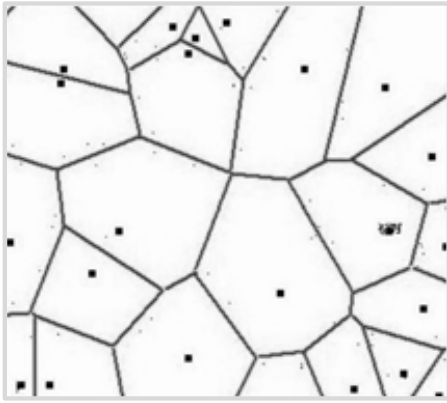


Fig. 4. Voronoi diagram (drawn by the authors)

smaller triangles (Fig. 5). The model begins with an equilateral triangle or square, and new triangles are generated by connecting the midpoints of each side. This process is repeated iteratively, creating a fractal structure (Bovill, 1996). The Sierpiński triangle offers both aesthetic and functional advantages in architectural design, particularly in modular systems, optimization of building components, and shading strategies.

#### Methods

The study employs a methodology combining qualitative and visual content analysis. Through qualitative content analysis, general information about the architectural characteristics of the selected buildings was obtained. Visual content analysis involved the examination of architectural drawings and photographs sourced from various websites. The buildings were evaluated in terms of their facades, plans, and architectural elements, with particular attention to five mathematical models and their relationship with nature. The results of these evaluations are presented in tables for each building. These findings are significant in demonstrating how the relationship between mathematics and nature is applied in architecture and the types of designs it inspires. More specifically, the purpose of applying visual content analysis was to determine how mathematical models inspired by nature are reflected in architectural design. This method made it possible to identify and classify the mathematical principles embedded in the form, plan, and facade

design of the buildings, thereby presenting the role of mathematical models in architectural practice in a more systematic manner. Furthermore, the process has the potential to generate datasets that may be integrated into future AI-supported content recognition and manual classification systems.

The buildings examined in this study are the Villa at Garches (Le Corbusier), Arab World Institute (Jean Nouvel), Toronto Engineering School (ZAS Architects), Beijing National Aquatics Center (PTW Architects + CSCE + Arup), and Grand Egyptian Museum (Heneghan Peng Architects). These iconic structures were selected because they exemplify both the application of mathematical principles and a design philosophy in harmony with nature. The Villa at Garches demonstrates the use of the golden ratio, aligning with human scale and natural context. The Arab World Institute incorporates complex fractal patterns inspired by Islamic geometry and environmental factors. The facade of the Toronto Engineering School is shaped by intricate tessellations, reflecting a strong relationship with nature. The Beijing National Aquatics Center has a facade structure based on Voronoi diagrams, effectively embodying natural processes and forms. Finally, the Grand Egyptian Museum integrates Sierpiński triangles into its plan, facade, and architectural elements, forging strong connections with both nature and cultural heritage. Due to their close ties with nature and mathematical principles, these buildings form a suitable dataset for the study.

The scope of the study is illustrated in the flowchart presented in Fig. 6.

#### Results

In this section, the findings derived from the case study buildings are summarized in tabular form (Tables 1–6).

##### *Villa at Garches*

Also known as Villa Stein-de Monzie, the Villa at Garches was designed by Le Corbusier in France between 1927 and 1928. It is one of his most iconic works, exemplifying five key architectural principles: pilotis, horizontal ribbon windows, a free facade, an open plan, and a roof garden. The building is characterized by spaciousness, flexibility, and modern functionality. Importantly, Le Corbusier considered the golden ratio not merely as

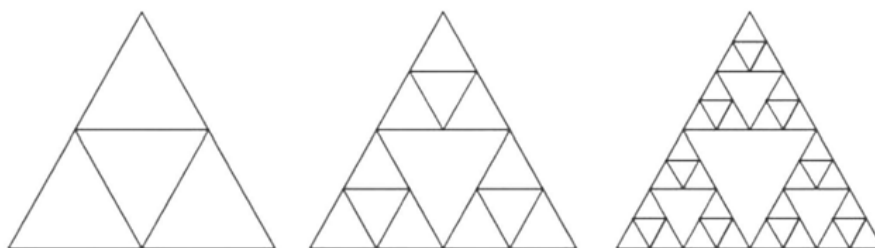


Fig. 5. Sierpiński triangle (drawn by the authors)

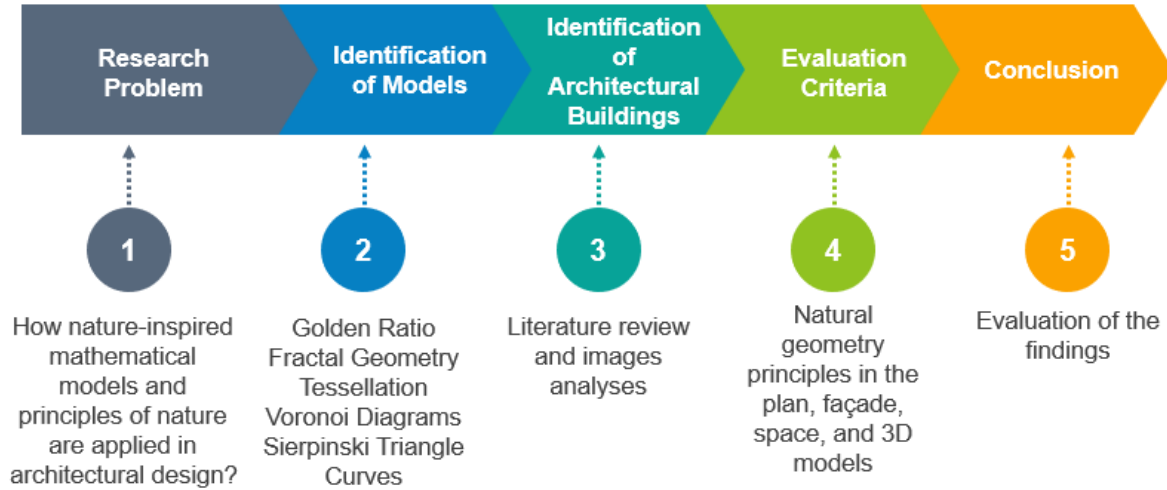


Fig. 6. Flowchart of the study

Table 1. Villa at Garches — building analysis

|                                                                                                                                                                                                                                                                                                                                    |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |
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| <p>(drawn by the authors)</p>                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |
| <b>Relation of the facade to the golden ratio</b><br><p>The harmony of horizontal and vertical facade proportions is organized according to the principles of the golden ratio. The placement of the windows, the spacing of the columns, and the overall dimensions of the facade are designed in accordance with this ratio.</p> | <b>Relation of the plan to the golden ratio</b><br><p>The width, length, and height of the spaces are proportioned based on the golden ratio. This ensured that the rooms and spaces are harmonious, rhythmic, and fluid.</p>                                                                                                     | <b>Relation of the architectural elements to the golden ratio</b><br><p>The stairs, window frames, columns, balcony projections, and eaves were designed according to the golden ratio principles.</p>                                                                                                                                                              |
| <b>Relation of the facade to nature</b>                                                                                                                                                                                                                                                                                            | <b>Relation of the plan to nature</b>                                                                                                                                                                                                                                                                                             | <b>Relation of the architectural elements to nature</b>                                                                                                                                                                                                                                                                                                             |
| <p>(en.wikiarquitectura.com, 2017; Front Desk, 2014; Wood, 2025)</p>                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                   |                                                                                                                                                                                                                                                                                                                                                                     |
| <p>The windows are designed to optimize daylight. The rhythm of the facade masses resembles the layering and depth found in natural forms. The large windows visually integrate the exterior with the interior.</p>                                                                                                                | <p>The building's plan presents a seamless integrity comparable to the fluidity observed in natural systems. The transitions between the interior and exterior spaces are continuous. Nature views are incorporated as active components of the interior, while the terraces strengthen the integration with the environment.</p> | <p>The thin delicate columns resemble tree trunks, while the openings between them allow the circulation of air and light like tree branches. The roof terrace establishes a direct interaction with nature. The continuity between the interior and exterior spaces, along with the inclusion of landscape elements on the roof, reinforces this relationship.</p> |

a mathematical principle but as a means to achieve aesthetic coherence and visual harmony. Through this application, the villa presents a design that is both balanced and visually satisfying (Herz-Fischler, 1984) (Table 1).

**Arab World Institute**

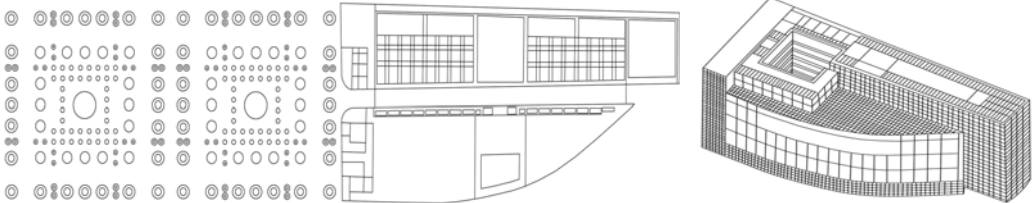
Designed by Jean Nouvel and completed in Paris in 1987, the Arab World Institute exemplifies a synthesis of modern and traditional design elements. Its most distinctive feature is the facade, composed of metallic panels that form a secondary wall. These panels are inspired by traditional mashrabiya motifs. By merging modern and traditional elements, the building seeks to create

a symbolic bridge between Arab and Western cultures. Functionally, the Arab World Institute serves as a multidisciplinary research and cultural center, accommodating educational, cultural, and commercial activities. Architecturally and culturally, it has become an iconic landmark within its urban context (McKiernan, 2013) (Table 2).

**Toronto Engineering School (Bergeron Center)**

The Toronto Engineering School was designed by ZAS Architects in 2018 for the Faculty of Engineering at the University of Toronto, Ontario, Canada. The building is distinguished by its modern and dynamic facade design. It incorporates sustainable features,

Table 2. Arab World Institute — building analysis

| <div></div> <div>(drawn by the authors)</div>                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                     |
|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Relation of the facade to fractals                                                                                                                                                                                                                                                                                                                                                       | Relation of the plan to fractals                                                                                                                                                                                                                                                                                                                                                                                                                                              | Relation of the architectural elements to fractals                                                                                                                                                                                                                                                                                                  |
| The traditional motifs on the facade exhibit fractal characteristics, consisting of repeating patterns across multiple scales. The circular motifs intertwine to create a complex, multi-layered facade. The interplay of light and shadow follows fractal geometry principles, and the repetition of patterns at macro and micro scales generates a dynamic and rhythmic visual effect. | The building plan organizes various functional spaces symmetrically around a central axis. The layout expands outward from the center according to fractal principles, with each unit reflecting the larger order at smaller scales. The interior spatial hierarchy repeats at multiple scales, consistent with fractal geometry principles.                                                                                                                                  | Numerous structural elements such as the shutters, interior columns, railings, as well as ceiling and floor coverings display fractal features. These elements contribute to a sense of balance, harmony, depth, and movement within the spaces. The facade shutters regulate daylight while producing fractal-like variations of light and shadow. |
| <div></div> <div>(Architecturestudio, 2025; cdn.sortiraparis.com, 2025; lookphotos, 2008)</div>                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                                                                                     |
| Relation of the facade to nature                                                                                                                                                                                                                                                                                                                                                         | Relation of the plan to nature                                                                                                                                                                                                                                                                                                                                                                                                                                                | Relation of the architectural elements to nature                                                                                                                                                                                                                                                                                                    |
| The facade design mimics dynamic natural processes. Traditional mashrabiya adapt to environmental conditions like organisms orient toward sunlight. Similar to plants opening and closing leaves for photosynthesis, the shutters adjust to control daylight, heating, and cooling. They also generate light patterns reminiscent of sunlight filtering through tree branches.           | The building plan represents an architectural expression of organic forms and cycles observed in nature. The central courtyard functions as an inner garden, facilitating natural ventilation, daylight penetration, and energy efficiency. The plan exhibits a multi-layered, integrated system in which different functional spaces interact similarly to natural processes. Continuity between the interior and exterior spaces maintains a strong connection with nature. | Natural forms and patterns inform the design of the architectural elements. Natural materials are used for the flooring and walls, and the interior spaces incorporate water features and plants to create a microclimate that reinforces a connection with nature.                                                                                 |

including a green roof, solar panels, and rainwater collection systems. The exposed structural systems transform the educational spaces into experimental environments. Overall, the Toronto Engineering School exemplifies a modern, innovative, sustainable, and flexible architectural approach. Environmentally responsive design strategies are a prominent feature of the building (URL-1) (Table 3).

#### **Beijing National Aquatics Center (Water Cube)**

Designed by PTW Architects, CSCE, and Arup and completed in 2008, the Beijing National Aquatics Center is located in Beijing, China. Its outer shell is inspired by the cellular structure of water molecules and the formation of foams in nature. Constructed for the 2008 Beijing Olympics, the building is recognized as an innovative and iconic example of contemporary architecture. It serves not only for competitive swimming but also for a variety of water

sports. In 2010, the addition of a water park further expanded its use (Zou and Leslie-Carter, 2010) (Table 4).

#### **Grand Egyptian Museum**

The design by Heneghan Peng Architects was selected as the first-place winner among 1,557 entries in an international competition. The construction began in 2005, and the museum officially opened in 2023. Covering a total area of 480,000 square meters, it is one of the largest archaeological museums in the world. Its design integrates references to Egypt's cultural heritage with a contemporary architectural approach. The building is situated near the Pyramids of Giza. The museum's triangular and sloping surfaces are inspired by the pyramids themselves. In addition to its primary exhibition function, the museum also serves as a cultural and scientific center. The building is

Table 3. Toronto Engineering School — building analysis

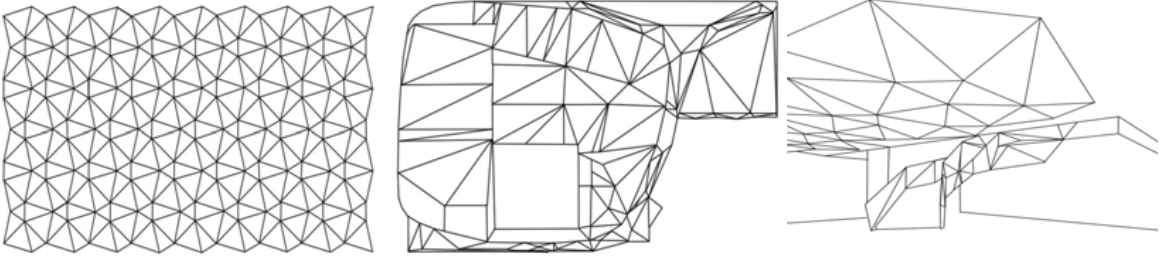
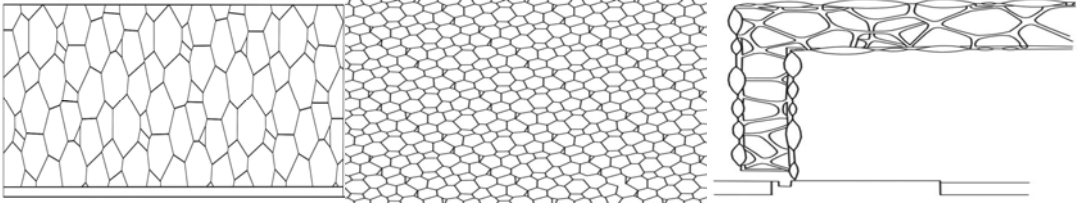
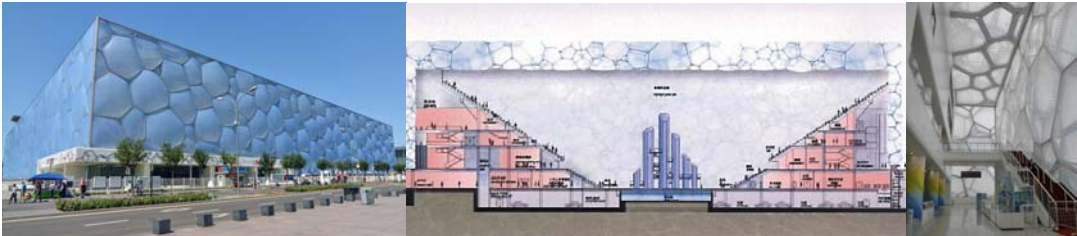
|  <p>(drawn by the authors)</p>                                                                                                                                |                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                        |
|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Relation of the facade to tessellation                                                                                                                                                                                                           | Relation of the plan to tessellation                                                                                                                                                                                                                                          | Relation of the architectural elements to tessellation                                                                                                                                                                                                                                 |
| The facade cladding is designed according to the tessellation principles. Glass and metal panels repeat rhythmically along the facade. Beyond their aesthetic role, these panels contribute to daylight control, energy efficiency, and shading. | The tessellation principles are applied in the interior design. Tessellations in classrooms, laboratories, and open offices enhance spatial efficiency, and promote seamless, fluid circulation throughout the building.                                                      | Tessellation patterns are integrated into the ceiling panels, floor and wall coverings, and railings. These patterns add depth and dynamism to the spaces. The lighting elements are also designed in accordance with the tessellation principles.                                     |
|  <p>(APA Facade Systems, 2025; images.adsttc.com, 2025)</p>                                                                                                  |                                                                                                                                                                                                                                                                               |                                                                                                                                                                                                                                                                                        |
| Relation of the facade to nature                                                                                                                                                                                                                 | Relation of the plan to nature                                                                                                                                                                                                                                                | Relation of the architectural elements to nature                                                                                                                                                                                                                                       |
| The dynamic facade reflects patterns found in nature. The cladding system mimics natural shadow play and evokes leaf-like motifs. The curved and flexible structure of the facade embodies the fluidity of organic forms.                        | The building plan evokes intertwined natural organisms. The functional spaces are interconnected to reflect symbiotic relationships in nature. Natural ventilation and daylight penetration are maximized, and the building is sited in harmony with the existing topography. | Natural materials, such as stone and wood, are employed to create a warm interior environment. Shadow patterns generated by the facade panels reinforce a connection with natural phenomena. The terraces and courtyards incorporate vegetation to enhance the connection with nature. |

Table 4. Beijing National Acuatics Center — building analysis

| <div></div> <div>(drawn by the authors)</div>                                                                                                                                                                                                                                                                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Relation of the facade to Voronoi diagrams                                                                                                                                                                                                                                                                                                                                                          | Relation of the plan to Voronoi diagrams                                                                                                                                                                                                                                                                                                                                                                                                                 | Relation of the architectural elements to Voronoi diagrams                                                                                                                                                                                                                                                      |
| The building's facade is inspired by Voronoi diagrams. The facade cells, arranged in an irregular organic pattern, are covered with ethylene-tetrafluoroethylene (ETFE) panels. Their arrangement according to Voronoi patterns creates dynamic light reflection and diffraction within the interior, producing a unique texture, multi-layered visual effects, and a sense of continuous movement. | The interior layout is also guided by Voronoi principles. The building plan organizes various functional spaces around a central core. The swimming pools and audience seating areas serve as primary functions, while service, circulation, and technical zones occupy supporting roles. The swimming pools and audience seating areas are designed according to Voronoi principles, promoting fluidity and efficient circulation throughout the space. | The structural elements and cladding also reflect Voronoi patterns. The cellular structure of these patterns is also maintained in the steel frame system, resulting in more balanced and stable load distribution throughout the structure.                                                                    |
| <div></div> <div>(Trubiano, 2013; w.litour.cn, 2025; Wordpress, 2025)</div>                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                                                                                                          |                                                                                                                                                                                                                                                                                                                 |
| Relation of the facade to nature                                                                                                                                                                                                                                                                                                                                                                    | Relation of the plan to nature                                                                                                                                                                                                                                                                                                                                                                                                                           | Relation of the architectural elements to nature                                                                                                                                                                                                                                                                |
| The facade draws inspiration from the geometric structures of water droplets and foams. These naturally irregular yet interconnected structures informed the design of the facade shell. The resulting cellular arrangement reflects the geometry, flexibility, and transparency of biological cell membranes.                                                                                      | The building plan evokes the fluidity and continuity of water, reflecting the symbiotic relationships observed among natural organisms. The building's interior is organized around the cooperative interaction of different functions, similar to an ecosystem. The water-related spaces are centrally located, reflecting the natural concept of gathering around a water source.                                                                      | The ETFE panels emulate the lightness, flexibility, and strength of cell membranes. The transparency of the panels maximizes natural daylight penetration. They also provide temperature regulation and natural ventilation of the interior. This contributes to energy efficiency and sustainable performance. |

distinguished by its integration of natural forms with cultural heritage and modern architecture, resulting in an original design (Attia et al., 2021) (Table 5).

The use of mathematical models and the relationship of the examined buildings with nature are summarized in Table 6. This table compares the buildings in terms of the mathematical models applied. As shown, nature-inspired mathematical models not only enhance aesthetic and structural optimization but also improve the integration of buildings with their surroundings. Such analyses are essential for understanding how mathematical models can be applied at different scales and incorporated into architectural design.

Conclusion

The relationship between mathematics and architecture in creating aesthetically pleasing and

functional structures is based on nature-inspired mathematical models, such as the golden ratio, Voronoi diagrams, the Sierpiński triangle, curves, and parabolas. These mathematical principles, derived from observations of nature, have enabled architects to design structures that are simultaneously environmentally friendly, durable, aesthetically balanced, structurally sound, and lightweight.

Each mathematical model contributes to architecture in distinct ways. The golden ratio is employed to achieve proportionally balanced and visually harmonious building forms. Fractal geometry establishes relationships among complex, self-repeating patterns, supporting the design of dynamic and organic spaces. Tessellations are used to model intricate repeating patterns, ranging from traditional ornamentation to modern facade cladding. Voronoi

Table 5. Grand Egyptian Museum — building analysis

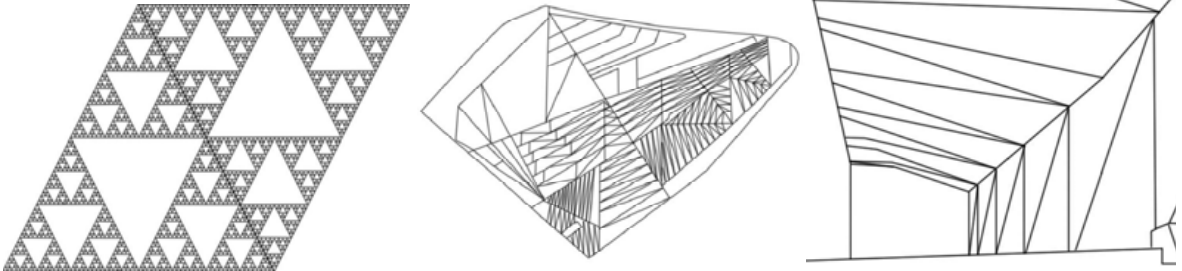
|  <p>(drawn by the authors)</p>                                                                                                                                                                                                                                                                                                      |                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| Relation of the facade to the Sierpiński triangle                                                                                                                                                                                                                                                                                                                                                                     | Relation of the plan to the Sierpiński triangle                                                                                                                                                                                                                                                                                                                       | Relation of the architectural elements to the Sierpiński triangle                                                                                                                                                                                                                                                                                                                                                                                               |
| The building's facade is inspired by Sierpiński triangles and curved surfaces. The multi-layered, repetitive triangular structure is clearly articulated, simultaneously referencing the nearby Pyramids of Giza.                                                                                                                                                                                                     | The Sierpiński triangle served as a reference for the museum's layout and interior design. Triangular and polygonal forms were employed in the planning of the exhibition spaces, research centers, and other functional areas, maintaining the hierarchical structure characteristic of Sierpiński triangles.                                                        | Sierpiński triangles are incorporated throughout the interior spaces, structural elements, and roof components of the building. The stairs and walkways consist of repetitive triangular units, and triangular patterns are also applied to the facades of the exhibition halls.                                                                                                                                                                                |
|  <p>(besix.com, 2025; Egypt Forward, 2025; orascom.com, 2025)</p>                                                                                                                                                                                                                                                                  |                                                                                                                                                                                                                                                                                                                                                                       |                                                                                                                                                                                                                                                                                                                                                                                                                                                                 |
| Relation of the facade to nature                                                                                                                                                                                                                                                                                                                                                                                      | Relation of the plan to nature                                                                                                                                                                                                                                                                                                                                        | Relation of the architectural elements to nature                                                                                                                                                                                                                                                                                                                                                                                                                |
| Inspired by the slopes and geometric structure of the Giza Pyramids, the building harmonizes with its natural and cultural surroundings. The curved surfaces of the facade allow daylight to enter the interior spaces at varying angles, introducing rhythm and dynamism reminiscent of natural processes. Additionally, the triangular forms on the facade echo crystalline and mineral structures found in nature. | The building's plan evolves in a manner similar to the growth of cellular structures in nature, exhibiting gradual and organic development. The circulation spaces feature soft and fluid transitions, reflecting the continuity and fluidity observed in natural systems. The layout of the exhibition spaces is additionally inspired by geological stratification. | The museum's structural system exhibits an organic design. The interior patterns and motifs are inspired by natural forms. The wall surfaces feature designs reminiscent of ancient Egyptian hieroglyphs. Stone and other natural elements are selected to harmonize with the environment. Additionally, natural landscaping and water features are incorporated into both interior and exterior spaces to provide cooling and enhance the sense of relaxation. |

Table 6. Study summary

| Building                         | Mathematical model  | Application in design                    | Relation to nature                                                                                                                           |
|----------------------------------|---------------------|------------------------------------------|----------------------------------------------------------------------------------------------------------------------------------------------|
| Villa at Garches                 | Golden ratio        | Facade, plan, architectural elements     | The spatial organization resembles the fluid structure of nature. The large windows enhance integration with the surrounding environment.    |
| Arab World Institute             | Fractal geometry    | Facade, plan, light control              | Light-shadow dynamics are generated through traditional mashrabiya motifs, imitating variable lighting effects observed in nature.           |
| Toronto Engineering School       | Tessellation        | Facade cladding, interior design         | The geometric patterns on the facade echo honeycomb structures.                                                                              |
| Beijing National Aquatics Center | Voronoi diagram     | Facade, structural system, light control | The outer shell mimics the irregular yet interconnected structure of water molecules.                                                        |
| Grand Egyptian Museum            | Sierpiński triangle | Facade, plan, structural elements        | The triangular forms reference the nearby pyramids, while the geometric patterns reflect crystalline and mineral structures found in nature. |

diagrams reflect cellular structures in nature and are applied in architecture for various purposes from the efficient and organic organization of spaces to facade cladding. The Sierpiński triangle facilitates complex visual textures, structural stability, and surface optimization. Curves and parabolas are employed in architecture to achieve both aesthetic refinement and engineering efficiency. Collectively, these nature-inspired mathematical models enrich architectural design by providing both aesthetic appeal and functional innovation.

Architects are increasingly applying mathematical models to develop innovative and sustainable designs. The advancement of computational design tools facilitates this process, creating new opportunities. Integrating mathematics allows for the creation of buildings that emulate the organic, self-organizing systems observed in nature. The method

employed in this study enabled the systematic analysis of nature-inspired mathematical models in architecture. Such approaches are valuable for structuring architectural design processes and for integration with digital design tools. With the ongoing development of artificial intelligence, content identification and coding can be made more efficient, reducing the need for manual classification and cataloging. Future studies may incorporate AI-supported systems to enable more comprehensive and automated evaluation of nature-inspired architectural designs. In conclusion, the evolving relationship between architecture and mathematics presents numerous opportunities for future advancements. This paper provides a comprehensive and contemporary analysis of the application of nature-inspired mathematical models and principles in architectural design.

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## АРХИТЕКТУРНЫЕ ПРОЕКТЫ, ВДОХНОВЛЕННЫЕ ПРИРОДОЙ И МАТЕМАТИЧЕСКИМИ МОДЕЛЯМИ

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### Аннотация

**Введение:** в данной статье рассматривается применение математических моделей, вдохновленных природой, а также принципов, наблюдаемых в природе, в архитектурном проектировании и их вклад в разработку инновационных и рациональных решений. Архитектурные сооружения, задействующие математические модели, вдохновленные природой, широко изучаются в литературе. Однако исследований, рассматривающих связь между природой и математикой именно в контексте архитектуры, все еще сравнительно немного. С тем чтобы закрыть данный пробел, в данной статье исследуется взаимодействие математики и природы в архитектурном проектировании. В рамках работы выполнен комплексный анализ применения математических моделей в архитектуре. Были проанализированы пять примеров, в основу которых положены различные математические принципы. **Методы:** сооружения оценивались путем качественного контент-анализа, а математические модели выявлялись с помощью визуального контент-анализа. Полученные результаты представлены в виде таблицы. **Заключение:** архитектура формируется под глубоким влиянием взаимодействия природы и математики. Анализ математических закономерностей, присущих природе, и их применение в проектировании служат ориентиром для создания эстетически привлекательных, экологически рациональных и инновационных зданий.

**Ключевые слова:** математическая модель; архитектура, вдохновленная природой; золотое сечение; фрактальная геометрия; математика в архитектуре.