DOI: 10.23968/2500-0055-2017-2-3-43-51

CALCULATION OF TANGENTIAL FROST HEAVE STRESSES BASED ON PHYSICAL, MECHANICAL AND STRESS-STRAIN BEHAVIOR OF FROZEN SOIL

Olga Tretiakova

Perm National Research Polytechnic University,
Komsomol'skii prospekt 29, Perm, 614990, Russia

Olga_wsw@mail.ru

Abstract

Development of northern territories opens up new opportunities for social and economic development of the society. However, designing construction facilities in such areas is complicated by special features of the severe climate; frost heave of soils is one of its manifestations. Normal frost heave forces under the foundation base can be neutralized by foundation embedment below the seasonal frost line. In these conditions, tangential heave stresses will act on the lateral surfaces of foundations, causing uneven lifting of structures, which will lead to the violation of the structure integrity. Regulatory and technical literature recommends determination of tangential frost heave stresses on the basis of generalized tabular data or experimentally, which is not always possible.

A relevant technique for calculating tangential stresses is required, which takes into account climatic and hydrogeological conditions of the construction site. In this article, an attempt is made to reveal those aspects of originating and development of tangential frost heave stresses, which would allow carrying out their calculation. To solve this problem, the existing research experience, features of the cryogenic structure, mechanical and stress-strain behavior of frozen soils were analyzed. A relationship of tangential frost heave stresses with specific cohesion and moisture of frozen soil was revealed on the basis of the analysis.

A point of view, according to which tangential frost heave stresses are a result of maximum compressive heave stresses, directed along the normal to the lateral surface of the foundation, was emphasized too; this allowed relating tangential heave stresses with the frozen soil deformation modulus. Based on the above statements, formulas for calculating tangential frost heave stresses were obtained. The calculation method for determining these stresses would provide the possibility of designing cost-effective and safe structures in areas with the severe climate.

Keywords

Wetness, boundary layer, tangential stresses, deformation modulus, frost heave, specific soil cohesion.

Introduction

In cold regions at the seasonal decrease of air temperature, cooling and freezing of the soil take place, which lead to increase in its volume, i.e. frost heave. Upon limitation of the surface expansion of foundations and embedded structures, normal stresses arise in soils that cause development of forces, which are tangential to these surfaces. Up to date, tangential frost heave stresses, acting on embedded structures, are determined either by generalized tables in regulatory documents, or experimentally. However, the latter is not always possible.

A question arises about the technique of calculating stresses for practical designing. Development of the calculation method, first of all, requires a study of the mechanism of originating and development of specified force factors. According to the existing concepts, tangential frost heave stresses develop upon freezing-up and subsequent interaction of the soil with the foundation material. However, understanding of heterogeneity of interacting media, which are the foundation and the soil, requires the development of these ideas and possible introduction of additional provisions in the definition of the stress-strain state of the "foundation–freezing soil" system. Theoretical prerequisites for this study are modern concepts of the cryogenic structure, mechanical and stress-strain behavior of the frozen soil, as well as existing concepts of the tangential stresses development staging.

The loading factor is an increase in soil volumes at freezing, leading to the formation of a stressed zone around the foundation. The effect of impact of the soil, expanding during freezing in this zone, are normal stresses, which, in their turn, cause stresses and shear forces directed tangentially to the foundation lateral surface.

The following tasks were set: to determine in which layer of the stressed zone shear stresses develop, leading to the development of tangential forces; this calculated layer will determine the qualitative and quantitative picture of the interaction; to reveal properties of interacting media, which would allow calculating tangential stresses; to establish a relationship of the stress value with mechanical and stress-strain properties of media. This would allow obtaining a technique for calculating stresses.

Background

In northern regions, dangerous natural process of frost heave of soils is one of manifestations of the severe climate. Normal stresses and stresses, tangential to foundation surfaces, develop under the influence of frost heave. Tangential stresses cause uneven lifting of structures, leading to their integrity violation and failures of buildings and infrastructure facilities. The influence of tangential stresses on structures is noted in works of B. I. Dalmatov (1954), V. D. Karlov (1998), M. F. Kiselev (1971), A. L. Nevzorov (2000), R. Sh. Abzhalimov (2006), D. Ladanyi (1998), J. P. Modisette (2014), and T. Kibriya (2015). Consideration of frost heave effect is a required condition for designing safe buildings and structures. Analysis of technical literature (https://ohranatruda.ru/ot_biblio/normativ/data_normativ/46/46329/) has shown that at the present time there is no generally accepted technique in regulatory documents for calculating tangential frost heave stresses, which is the reason to use recommended tabular values in practical designing or determinate stresses experimentally. However, tabular data do not allow taking into account the entire range of soil and climatic conditions, as well as their possible combinations. These data were obtained by results of experiments, which were carried out for a limited number of soils. Experimental determination of tangential heave stresses on a particular site is not always possible due to limited terms of execution of project documentation and amounts of financing. Experimental estimation, which provides for full-scale or laboratory researches, extends designing terms and implies for an additional item of expenditures for surveying and designing works. A full-scale experiment requires a term, corresponding to the period of negative temperatures of the atmospheric air, tools for measuring tangential heave stresses and special trained technical personnel. Methods of field measuring of tangential stresses are represented in works of B. I. Dalmatov (1954), V. D. Karlov (1998) N. A. Tolkachev (1964), E. A. Marov (1974), E. D. Ershov (1986), A. G. Alekseev (2006), H. Jiang (2015). A laboratory experiment is impossible without proper premises, special certified equipment and scientific and technical maintenance. Methods of the laboratory determination of frost heave forces are shown in the Guidelines (1973). Researches on frost heave on models in laboratory conditions were conducted by E. D. Ershov (1985), V. R. Parmesvaran (1981), Thomas H. (2009), Y.-H. Hyang (2015), and F. Ming (2015).

A question regarding effectiveness of the experimental approach, more inherent to scientific research, to prediction of tangential frost heave stresses during practical designing arises. The use of the approved method for calculating tangential frost heave stresses, based on fundamental studies of frost heave, would allow designing safe structures.

In Russia, the basic theory of frost heave, including the description of normal and tangential stresses, was developed by N. A. Tsytovich (1973), B. I. Dalmatov (1988), and S. S. Vyalov (1959). Abroad, the physics of frost heave is presented in works of R. L. Harlan (1973), J.-M. Konrad (1980), and S. S. L. Peppin (2012). E. Penner (1974, 2010), S. Frankenstein (2002), K. W. Biggar (2011), and S. Hiroshi (2011) were engaged in determining values of tangential stresses and frost heave forces. According to the existing concepts, the formation of tangential stresses is determined by forces of soil adfreezing with the lateral surface of the foundation. Heave forces, which develop in the surrounding soil mass, tend to move the foundation upwards. At that, frozen soil shear relative to the foundation occurs. Static bonds of the soil adfreezing with the foundation (adfreezing strength) are violated. Dynamic bonds arise which are determined by the resistance to the displacement of the frozen soil layer relative to the lateral surface of the foundation — tangential heave forces.

According to the existing studies, the value of tangential frost heave forces is close to values of steady forces of soil adfreezing with the foundation material. This provision was represented in works by B. I. Dalmatov (1988), N. A. Tsytovich (1973) and S. S. Vyalov (1959). Adfreezing forces are understood as the resistance to frozen soil shear throughout the lateral surface of the foundation. The force of soil adfreezing with the foundation corresponds to the general force which is needed to be applied to it in order to break the bond with the soil frozen around. Thus, it is considered that displacement of the frozen soil relative to the foundation takes place between the foundation material and the soil.

The resistance to frozen soil shear is a function of main variables, which are physical, thermal physical and mechanical properties of the soil, characteristics of the foundation material. Properties of frozen soil are conditioned by its complex cryogenic structure, which, in its turn, is determined by the temperature, freezing rate, external load value and time of its action. Characteristics of the foundation material are related to the characteristics of roughness of its surface. The listed factors indicate heterogeneity of media interacting during shear. Whereas, for shear occurrence directly between the material and frozen soil, these two media shall be homogeneous. All this creates prerequisites for the development of existing provisions on the determination of tangential frost heave stresses.

DOI: 10.23968/2500-0055-2017-2-3-43-51

Theoretical prerequisites of the study

The analysis of the existing works devoted to cryogenic structures, mechanical properties and stress-behavior of frozen soil was carried out. As N. A. Tsytovich (1973) points out, frozen soil represents a four-component system consisting of soil particles, inclusions of ice, unfrozen film water and gaseous inclusions. The amount and phase states of components in the combination with natural properties of the soil determine its heterogeneous cryogenic structure. In this structure, soil particles are "aggregated" by ice-cement. The contact surfaces of particles of the soil and ice form numerous shear planes, which provide the possibility of frozen soil brittle fracture from shear stresses during shear.

Studies of the values of tangential frost heave stresses were carried out by B. I. Dalmatov, Y. D. Dubnov, N. A. Tsytovich, S. S. Vyalov and others. B. I. Dalmatov found a linear dependence of steady adfreezing resistance on temperature (Figure 1), and, based on this, a formula for specific tangential heave force was obtained:

$$\tau_{heave} = \frac{h_{heave.}}{h}\left(c + b\, t_{mean}\right)$$

where *c* and *b* are parameters of the straight line, determined in laboratory conditions and depending on the soil type and its moisture, c = 4–5 t/m^2; b = 1–1.5 t/0C·m^2;

h_{heave} is thickness of the layer subjected to heave;

h is depth of freezing, m;

t_{mean} is mean temperature of the soil within the layer h_{heave} by the time of its freezing completion.

The non-uniformity of tangential stresses development over time is shown by N. A. Tsytovich (1973), S. S. Vyalov (1959), B. I. Dalmatov (1988), and V. I. Puskov (1993).

Based on the current concepts of staging of the tangential stresses development, the author considered the work of interacting media. At the first stage, shear with soil mass isolation occurs relative to the thin boundary layer of this soil, located on the lateral surface of a pile and frozen together with it. In this case, tangential shear stresses have a significant value, but a small period of activity. These stresses can be characterized as a short-term, almost instantaneous shear occurring when the soil reaches the ultimate resistance to shear.

In this case, ultimate shear strength of particles of the frozen soil boundary layer connected with the surface of the embedded structure, which is practically its "part", is of the paramount importance. Generation of tangential shear stresses is conditioned by overcoming total cohesion of frozen soil particles, corresponding to the ultimate resistance of the soil and ice to shear; normal pressure of the freezing soil on the lateral surface of the foundation.

The activity period of ultimate (instantaneous) tangential shear stresses τ_{inst}, causing shear with the isolation of soil aggregates in the boundary layer is replaced by the period of decay (stress relaxation), which continues up to the stabilization of the constant soil shear rate throughout the lateral surface of a pile at stresses, which reached the value of τ_{steady}. Further, the second stage of steady-state tangential stresses τsteady begins; these stresses appear when the frozen soil massif is uniformly displaced relative to the boundary layer. The massif displacement occurs due to micro-displacements in the boundary layer, in which a part of bonds (least strong) is already broken at the stage of short-term shear. Here tangential stresses have less importance, but they act during almost the entire freezing period and determine the actual picture of the development of tangential frost heave stresses throughout the lateral surface of the pile. This is confirmed by researches of B. I. Dalmatov (1958), N. A. Tsytovich (1973) and S .S. Vyalov (1959). A diagram of the dependence of shear resistance on time by B. I. Dalmatov is shown in Figure 1b.

At the second stage, relaxation of stresses from τ_{inst} to τ_{steady} occurs, mainly due to the reduction of frozen soil cohesion forces, under the action of the continuous load from frost heave forces. This phenomenon was described in experiments of S. S. Vyalov (1959).

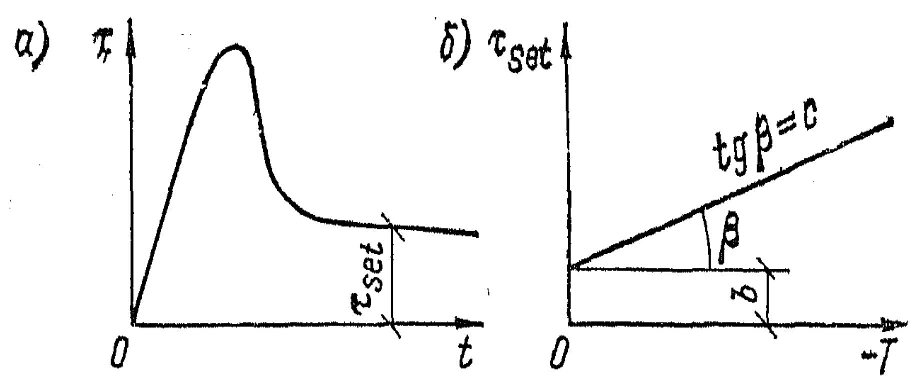

Figure 1. a) graph of dependence of shear resistance on time; b) graph of dependence of steady adfreezing resistance on soil temperature (B. I. Dalmatov, Y. D. Dubnov).

Characteristic change in cohesion of frozen soils over time, which forms the prevailing part of their shear resistance, is also illustrated in dependences of N. A. Tsytovich (1963). The studies, carried out by S. S. Vyalov (1959) and N. A. Tsytovich (1973), showed that cohesion forces (the main strength characteristic of frozen soil), decreased under the action of shear stresses over time, as a rule, within 1/3 –1/5 c_{inst}. Also, the internal friction angle decreases for clays at continuous shear.

Methods of calculation of tangential frost heave stresses

Special features of the cryogenic structure and mechanical properties of frozen soils allowed assuming that shear stresses caused by soil heave occur not along the "foundation–freezing soil" boundary, but in the boundary freezing layer adjacent to the foundation. This proposition is substantiated by aspects considered above, which are the theoretical prerequisites of this study: heterogeneity of interacting continuous media — the foundation material and freezing soil; and the ability of frozen soil for brittle fracturing.

The effect of constrained heave impact near the lateral surface of the foundation is represented by significant compressive forces, leading to the development of shear stresses, which are realized throughout numerous shear planes formed due to different shear strength of soil components in the calculated boundary layer. Mechanical properties of ice are analyzed in the work of V. V. Bogorodskiy (1983). Tangential forces are the resultant factor of these shear stresses. Violation of bonds between particles of the soil and ice is needed for origination of shear deformations. Shear in any elementary fragment will take place between ice layers or along the boundary of ice and mineral interlayers due to the lower shear strength of soil particles. In some approximation, it is possible to estimate the distribution of shear deformations according to the percent ratio between the soil and ice. This allowed establishing a relationship of the value of tangential frost heave stresses with the soil moisture, since the amount of ice inclusions in the frozen soil is determined primarily by the moisture of the latter; as well as with the specific cohesion of soil and ice particles. In view of the foregoing, the total cohesion of frozen clay-bearing soil at the first stage of the development of tangential stresses can be expressed in the following form:

$$c^{inst} = c^{inst}_{soil} \cdot (1 - 1.09 \cdot w) + c^{inst}_{ice} \cdot 1.09 \cdot w = c^{inst}_{soil} - 1.09 \cdot w \cdot c^{inst}_{soil} + c^{inst}_{ice} \cdot 1.09 \cdot w \quad (1)$$

After transformations, the following formula was obtained:

$$c^{inst} = c^{inst}_{soil} - 1.09 \cdot w \cdot \left(c^{inst}_{soil} - c^{inst}_{ice}\right) \quad (2)$$

where c^{inst}_{soil} is soil specific cohesion, corresponding to the ultimate resistance, kPa; c^{inst}_{ice} is ultimate ice resistance to shear, kPa; *w* is soil moisture, unit fraction.

As B. I. Dalmatov points out, shear stresses originating in cohesive clay-bearing soils near the lateral surface of the foundation, are the following:

$$\tau = c + \sigma \cdot tg\varphi \quad (3)$$

where *c* is specific cohesion; *σ* is normal pressure; *φ* is internal friction angle.

It is assumed that the dependence (3) can be used to determine tangential shear stresses of frost heave, developing on the lateral surface of foundations, using basic variables of B. I. Dalmatov's formula for the specific tangential frost heave force. The B. I. Dalmatov's formula for the specific tangential frost heave force has the following form:

$$\tau^{heave} = \frac{h_{heave}}{h}\left(c + b\,t_{mean}\right)$$

where *c* and *b* are parameters of the straight line, determined in laboratory conditions and depending on the soil type and its moisture, c = 4–5 t/m²; b = 1–1.5 t/0C·m²;

h_{heave} is thickness of the layer subjected to heave;

h is depth of freezing, m;

t_{mean} is mean temperature of the soil within the layer hheave by the time of its freezing completion.

Main variables are mechanical properties of the soil, moisture, temperature of the frozen soil, thickness of the freezing soil layer by the time of its freezing completion.

As per the above, tangential stresses at the first stage can be expressed as follows according to equation (3):

$$\tau^{I}_{\max} = c^{inst} + p_{mean} \cdot tg\varphi_1 \quad (4)$$

where c_{inst} is total (instantaneous) cohesion of frozen soil particles, corresponding to the ultimate resistance, kPa;

p_{mean} is mean pressure on the lateral surface of the foundation contacting with the frozen soil, kPa;

φ_i is angle of internal friction between the soil and ice.

The mean pressure on the lateral surface of the foundation has the following form:

$$p_{mean} = \xi_i \cdot \gamma_i \cdot z_i \quad (5)$$

where ξ_i is coefficient of the lateral pressure of the soil and ice.

$$\xi_i = \frac{v_i}{1 - v_i}$$

γ_i is volume density of the soil and ice, kPa;

z_i is depth (thickness) of ith layer of the frozen soil, m;

v_i is Poisson's ratio for the soil and ice.

Substituting formulas (2) and (5) into formula (4), we obtain the maximum tangential frost heave stress within the boundary layer at the depth z_i at the specified soil moisture. This instantaneous stress is the ultimate shear strength of the frozen soil.

DOI: 10.23968/2500-0055-2017-2-3-43-51

$$\tau^I_{max} = \tau^I_{inst} = \left[c^{inst}_{soil} - 1.09 \cdot w \cdot \left(c^{inst}_{soil} - c^{inst}_{ice}\right)\right] + \xi_{soil} \cdot \gamma_i \cdot z_i \cdot tg\varphi_{soil} \cdot (1 - 1.09 \cdot w) + \xi_{ice} \cdot \gamma_{ice} \cdot z_i \cdot tg\varphi_{ice} \cdot 1.09 \cdot w \qquad (6)$$

At ***the second stage***, steady-state tangential stresses develop, which are more reliable indices than the short-term ones at the first stage. Steady-state stresses are characterized by a decrease in shear stresses due to a decrease in the specific cohesion of the soil and angle of internal friction for cohesive soils, according to N. A. Tsytovich (1973) and S. S. Vyalov (1959). Based on this, it is possible to determine the steady-state stresses from equation (6), replacing the instantaneous cohesion c_{inst} of the soil by its continuous value c_{con}.

$$\tau^I_{steady} = \left[c^{con}_{soil} - 1.09 \cdot w \cdot \left(c^{con}_{soil} - c^{con}_{ice}\right)\right] + \xi_{soil} \cdot \gamma_i \cdot z_i \cdot tg\varphi_{soil} \cdot (1 - 1.09 \cdot w) + \xi_{ice} \cdot \gamma_{ice} \cdot z_i \cdot tg\varphi_{ice} \cdot 1.09 \cdot w \qquad (7)$$

Analyzing the moisture value ***w*** in equation (7), let us note that the ice formation in the freezing soil is conditioned by the total moisture content, including the pore moisture in the natural composition of the soil and the moisture entering as a result of migration.

$$w_\Sigma = w + w_{migr} \qquad (8)$$

Let us substitute the total moisture content (8) instead of ***w*** into equation (7):

$$\tau^I_{steady} = \left[c^{con}_{soil} - 1.09 \cdot (w + w_{migr}) \cdot \left(c^{con}_{soil} - c^{con}_{ice}\right)\right] + \xi_{soil} \cdot \gamma_i \cdot z_i \cdot tg\varphi_{soil} \cdot \left(1 - 1.09 \cdot (w + w_{migr})\right) + \xi_{ice} \cdot \gamma_{ice} \cdot z_i \cdot tg\varphi_{ice} \cdot 1.09 \cdot (w + w_{migr}) \qquad (9)$$

Let us determine the share of the migration moisture in the freezing soil, using the formula for heave deformation proposed by A. L. Nevzorov (2000). In this equation, the second summand is connected with the moisture entering the freezing zone due to migration forces:

$$h_{heave} = 0.09 \cdot (w_{sat} - w_w) \cdot \frac{\rho_d}{\rho_w} \cdot z + 1.09 \cdot SP \cdot \tau \cdot grad\ t \qquad (10)$$

Then, the moisture forming due to water migration to the cold front, according to equation (10), will be:

$$w_{migr} = \frac{SP \cdot \tau \cdot grad\ t}{z} \qquad (11)$$

After substitution of (11), equation (9) will take the following form:

$$\tau^I_{steady} = \left[c^{con}_{soil} - 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z}) \cdot \left(c^{con}_{soil} - c^{con}_{ice}\right)\right] + \xi_{soil} \cdot \gamma_i \cdot z_i \cdot tg\varphi_{soil} \cdot \left(1 - 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z})\right) + \xi_{ice} \cdot \gamma_{ice} \cdot z_i \cdot tg\varphi_{ice} \cdot 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z}) \qquad (12)$$

where c^{con}_{soil} is specific cohesion of soil particles, kPa;
c^{con}_{ice} is specific cohesion of ice particles, kPa;
w is natural soil moisture, unit fraction;
φ_{soil}, φ_{ice} are angles of internal friction of the soil and ice, respectively;
ξ_{soil}, ξ_{ice} are coefficients of the lateral pressure of the soil and an ice, respectively;
γ_{soil}, γ_{ice} are volume weights of the soil and ice, respectively, kN/m^3;
z is depth of soil freezing in the vertical direction; thickness of the soil layer in the horizontal direction, m;
SP is segregated soil potential, m^2/h·°C;
τ is freezing time, hours;
grad t is temperature gradient, 0C/m.

Let us simplify equation (12), substituting c^{con}_{soil} and c^{con}_{ice} by the values c_{soil} and c_{ice}, and τ^I_{steady} by τ_{steady}:

$$\tau = \tau_{steady} = \left[c_{soil} - 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z}) \cdot (c_{soil} - c_{ice})\right] + \xi_{soil} \cdot \gamma_i \cdot z_i \cdot tg\varphi_{soil} \cdot \left(1 - 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z})\right) + \xi_{ice} \cdot \gamma_{ice} \cdot z_i \cdot tg\varphi_{ice} \cdot 1.09 \cdot (w + \frac{SP \cdot \tau \cdot grad\ t}{z}) \qquad (13)$$

As it was noted above, main variables of the B. I. Dalmatov's formula for specific tangential heave strength are the following: mechanical properties of the frozen soil, moisture, temperature of the frozen soil by the time of its freezing completion, thickness of the freezing layer. As it was mentioned above, main variables of equation (13) are the following: mechanical properties of the frozen soil, moisture, temperature gradient within the frozen layer; freezing time, segregated soil potential (characterized by heave velocity, moisture migration rate and temperature gradient), and thickness of the freezing layer.

The reason of the generation of tangential stresses on the lateral surface of the foundation is maximum normal compressive stresses developing when limitating expansion of the heaving soil during its freezing. This proposition was used in obtaining the *method for calculating tangential frost heave stresses*. The relationship of tangential stresses with normal frost heave stresses was found by V. I. Puskov (1973). V. I. Puskov notes that "in the absence of a local load, on the soil surface in the layer of seasonal freezing, volume deformation *f* is completely realized in the direction of the vertical axis and includes, besides f_z, lateral heave deformations transformed in the direction of *z* axis by the arising lateral pressure σ_{fb}".

Stating methods for determining heave deformations, V. D. Karlov (1998) in his DSc thesis noted that "the considered methods also allow determining the value of specific forces of soil frost heave depending on the degree of constraint of heave deformations". And the degree of constraint of heave deformations, in its turn, conditions normal frost heave stresses, which can be traced in works of various authors. For example, according to N. A. Tsytovich (1973), "the order of the maximum value of normal frost heave forces can be estimated based on the values of pressures which ice crystals develop upon the constrained water freezing". According to E. D. Ershov, “stresses in freezing rocks are conditioned by the exclusion of heave deformations”. Thus, the existing studies allowed expressing tangential frost heave stresses with the use of the Coulomb law in dependence to horizontal normal frost heave stresses σ_x:

$$\tau = \sigma_x \cdot t\,g\varphi \tag{14}$$

where σ_x is frost heave stress normal to the lateral surface of the foundation according to the formula obtained by the author (2016); φ is angle of internal friction of the frozen soil or ice.

The author obtained the formula for calculating horizontal frost heave stresses σ_x(2016), considering properties of the frozen soil, depth (thickness) of the freezing layer and freezing conditions:

$$\sigma_x = \sigma_{heave} = 1.09 \cdot SP \cdot \tau \cdot grad\ t \cdot \frac{E_M}{z} \cdot \left[1 - e \cdot \left(1 - w_w \cdot \frac{\rho_d}{\rho_w} - 1.09 \cdot w \cdot \frac{\rho_d}{\rho_w}\right)\right] \cdot k_{an} \tag{15}$$

where E_M is modulus of frozen soil deformation; w is natural moisture of the soil; w_w is moisture by unfrozen water; e is porosity factor of the soil; z is depth of soil freezing in the vertical direction; thickness of the soil layer in the horizontal direction; *SP* is segregated soil potential; τ is freezing time; *grad t* is temperature gradient; ρ_d is density of the dry soil; ρ_w is density of free water; ρ_d / ρ_w is a factor of conversion of the mass moisture into the volume moisture; k_{an} is an anisotropy factor of the frozen soil, taking into account the direction of freezing and frost heave stresses.

Then equation (14) for tangential frost heave stresses will take the following form:

$$\tau = \left\{1.09 \cdot SP \cdot \tau \cdot grad\ t \cdot \frac{E_M}{z} \cdot \left[1 - e \cdot \left(1 - w_w \cdot \frac{\rho_d}{\rho_w} - 1.09 \cdot w \cdot \frac{\rho_d}{\rho_w}\right)\right] \cdot k_{an}\right\} \cdot t\,g\phi \tag{16}$$

Due to the unfrozen water, the soil moisture is determined as a function of the plastic limit moisture (in unit fractions), according to Regulations SP 25.13330.2012:

$$w_w = K_w \cdot w_p$$

where k_w is a factor taken from table B.3 [Regulations SP 25.13330.2012] in dependence to the index of plasticity and soil temperature; w_p is soil moisture at the plastic limit.

Kujala found that the segregated potential SP (mm^2/h·°C) was in correlation with the heave velocity h_v (mm/day):

$$SP = 1.1 \cdot v_h = mm^2 / hour \cdot {}^0C$$

where v_h is heave velocity, mm/day.

The mean temperature gradient for the layer *z* is calculated as the ratio of the mean winter temperature to 0.5 z.

$$grad\ t = \frac{t_{bf} - t_s}{0.5 \cdot z} \approx \frac{t_s}{0.5 \cdot z}\ {}^0C/cm$$

where t_{bf} is temperature at the beginning of freezing $t_{bf} \to 0$; t_s is mean winter temperature; *z* is thickness of the freezing edge (freezing layer).

The time of freezing amounts to:

$$\tau = \frac{z}{v_n}$$

where v_n is soil freezing velocity.

Equation (15) after the substitution of values will take the following form:

$$\sigma_x = 1.09 \cdot 1.1 v_h \cdot \frac{z}{v_n} \cdot \frac{t}{0.5z} \cdot \frac{E_M}{z} \cdot \left[1 - e \cdot \left(1 - w_w \cdot \frac{\rho_d}{\rho_w} - 1.09 \cdot w \cdot \frac{\rho_d}{\rho_w}\right)\right] \cdot k_{an} \tag{17}$$

After transformations we get the following:

$$\sigma_x = 2.4(m/{}^0C) \cdot \frac{v_h}{v_n} \cdot \frac{t \cdot E_M}{z} \cdot \left[1 - e \cdot \left(1 - w_w \cdot \frac{\rho_d}{\rho_w} - 1.09 \cdot w \cdot \frac{\rho_d}{\rho_w}\right)\right] \cdot k_{an} \tag{18}$$

Then, equation (16) will take the following form:

$$\tau = 2.4(m/{}^0C) \cdot \frac{v_h}{v_n} \cdot \frac{t \cdot E_M}{z} \cdot \left[1 - e \cdot \left(1 - w_w \cdot \frac{\rho_d}{\rho_w} - 1.09 \cdot w \cdot \frac{\rho_d}{\rho_w}\right)\right] \cdot k_{an} \cdot t\,g\varphi \tag{19}$$

It is rather difficult to determine the modulus of frozen soil deformation. In regulatory documents, it is recommended to determine this parameter experimentally. According to the recommendations of "Guidelines for Determining Physical, Thermal Physical and Mechanical Properties of Frozen Soils" (https://ohranatruda.ru/ot_biblio/normativ/data_normativ/48/48098/), the deformation modulus is the value reciprocal to the compressibility. The compressibility, in its turn, is determined depending on the volume weight ρ and soil moisture *w*.

Then the following basic physical values can be taken as parameters determining the value of tangential stresses in equation (19):

$$f(w, t, z, v_h, v_n, \rho, t\,g\varphi) = 0 \tag{20}$$

DOI: 10.23968/2500-0055-2017-2-3-43-51

Equation (20) makes it possible to determine similarity criteria of tangential stresses on the basis of methods of applying the similarity theory and dimension theory in the mechanics of frozen soils, which were stated in works of B. I. Dalmatov, V. D. Karlov, O. R. Golli and E. S. Ashpiz.

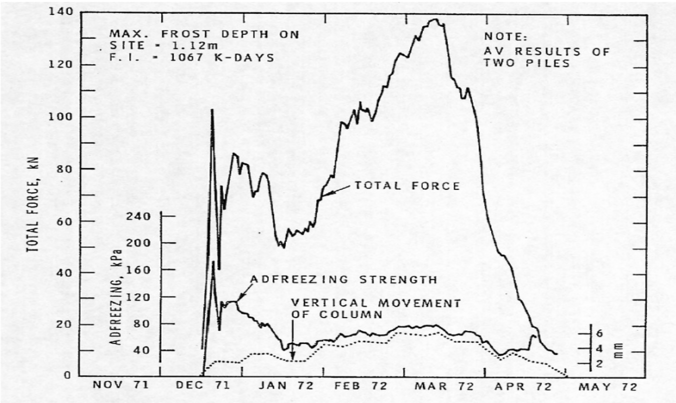

Figure 1. Part of matrix of physical laws as a canvas for analysis

Results

Based on equations (13) and (16), examples of calculating tangential heave stresses, acting on the lateral surface of the foundation in the clay-bearing soil are fulfilled. Despite some schematization, formulas give satisfactory results shown in Table 1. Comparison of stresses calculated by equation (13) and (16) with the values obtained by some other authors is presented in Table 2.

Table 1. Results of calculating tangential frost heave stresses

Equation No	Calculation formula	Value of tangential stresses, kN/m²
13	$\tau=\left[c_{soil}-1.09\cdot(w+\frac{S\,P\cdot\tau\cdot grad\,t}{z})\cdot(c_{soil}-c_{ice})\right]+$ $+\xi_{soil}\cdot\gamma_i\cdot z_i\cdot t\,g\varphi_{soil}\cdot\left(1-1.09\cdot(w+\frac{S\,P\cdot\tau\cdot grad\,t}{z}\right)+$ $+\xi_{ice}\cdot\gamma_{ice}\cdot z_i\cdot t\,g\varphi_{ice}\cdot1.09\cdot(w+\frac{S\,P\cdot\tau\cdot grad\,t}{z})$	152.7
16	$\tau=2.4(m/{}^0C)\cdot\frac{v_h}{v_n}\cdot\frac{t\cdot E_M}{z}\cdot\left[1-e\cdot\left(1-w_w\cdot\frac{\rho_d}{\rho_w}-1.09\cdot w\cdot\frac{\rho_d}{\rho_w}\right)\right]\cdot k_{an}\cdot t\,g\varphi$	79

Conclusion

In order to increase the effectiveness of designing of underground structures in cold areas, a technique for calculating tangential frost heave stresses is required.

Based on modern concepts of the cryogenic structure, mechanical properties and strain behavior of the frozen soil, a correlation of tangential frost heave stresses with moisture and specific cohesion of frozen soil particles is ascertained. Based on this, a formula is obtained for calculating tangential frost heave stresses (14).

The reason of the generation of tangential stresses is compressive heave stresses, normal to the lateral surface of the foundation and leading to shear. Formula (16) for tangential stresses calculation is proposed, as a function of the specified normal pressure. The latter was obtained by the author (2016) on the basis of physical properties and strain behavior of the frozen soil, freezing conditions, and regularities of moisture migration.

The values calculated according to formulas (13) and (16) do not contradict the data of other researchers.

Table 2. Comparison of tangential frost heave stresses

Author of the study	Type of the study	Freezing depth, m	Tangential heave stress, kN / m²	Mean tangential heave stress, kN / m²
O. V. Tretiakova (2017)	Calculation	1.0	79–152.7	115.9
		0–0.5	270–300	
		0–1.0	140–160	
		0–1.5	120–135	
		0–2.5	100	
		0–3.5 and >	80	
R. Sh. Abzhalimov (2006)	Field study	120		120
D. Ladanyi, A. Foriero Figure 2 (1998)	Field study	1.12	60–160	120

Discussion

Currently, issues of the uneven distribution of tangential stresses throughout the depth and variability of their values over time remain controversial.

Studies of the frozen soil deformation moduli will help to take into account the non-uniformity of tangential stresses throughout the depth. These indices can be determined in laboratory conditions. In Russia, strength properties and strain behavior of frozen soils were studied by L. T. Roman (2016), R. Sh. Abzhalimov (2009, 2011) and others. Abroad, these issues were analyzed in the works of T. F. Azmatch (2011), K. W. Biggar (2011) and others. The carried out studies are useful for the increase of applicability of the calculating formulas obtained by the author.

An important role in the development of calculation techniques belongs to rheological properties of the soil, which determine the unevenness of tangential stresses over time.

Long-term soil strength is more relevant for assessing the suitability of the construction safety for the entire lifetime than the short-term breaking ultimate strength limit obtained during tests. This area represents a field for further researches.

References

Abzhalimov, R.Sh. (2006). Opredelenie velichiny vypuchivaniia grunta vokrug svai, samikh svai i svainykh rostverkov, usilii v nikh na puchinistykh gruntakh [Determination of soil uplift gradient around piles and pile grids, their stresses on heaving soils]. Internet journal. *Razvitie gorodov i gidrotekhnicheskoe stroitel'stvo [City development and hydraulic engineering]*, 10, pp. 214–221. (in Russian)

Abzhalimov, R.Sh. (2009). Printsipy rascheta maloetazhnykh zdanii i podzemnykh sooruzhenii na puchinistykh gruntovykh osnovaniiakh [Calculation principles of low-storey buildings and underground structures on heaving soil foundations]. *Geotekhnika*, 1, pp. 25–33. (in Russian)

Abzhalimov, R.Sh. (2011). *Zakonomernosti vzaimodeistviia puchinistogo gruntovogo osnovaniia s fundamentami maloetazhnykh zdanii i podzemnymi sooruzheniiami i metody ikh rascheta [Regularities of the interaction of heaving soil foundations with foundations of low-storey buildings and underground structures and methods of their calculation]*. DSc Thesis in Engineering. Moscow: Moscow State University of Railway Engineering, p. 47. (in Russian)

Alekseev, A. G. (2006). *Opredelenie gorizontal'nogo davlenija grunta na podpornye steny pri sezonnom promerzanii-ottaivanii [Determination of frost heave horizontal pressure on retaining walls under seasonal freeze-thaw]*. PhD thesis in Engineering. Moscow: OJSC "PNIIIS", p. 190. (in Russian)

Ashpiz, E. S. (1984). *Uslovija podobija processov promerzanija i puchenija gruntov pri fizicheskom modelirovanii [Conditions of the similarity of freezing and heaving processes in physical modeling].* Dnepropetrovsk: Dnepropetrovsk Institute of Railway Transport Engineers named after M. I. Kalinin, p. 39. (in Russian)

Azmatch, T. F, Sego, D. C, Arenson, L. U, Biggar, K. W. (2011). Tensile strength and stress-strain behaviour of Devon silt under frozen fringe conditions. *Gold Regions Science and Technology*, 68(1–2), pp. 85–90. DOI: 10.1016/j.coldregions.2011.05.002

Biggar, K. W., Sego, D. (1993). The strength and deformation behaviour of model adfreeze and grouted piles in saline frozen soils. *Canadian Geotechnical Journal,* 30(2), pp. 319–337. DOI: 10.1139/t93-027

Bogorodskiy, V. V., Gavrilo, V. P., Nedoshivin, O. A. (1983). *Razrushenie l'da. Metody, tekhnicheskie sredstva [Ice destruction. Methods, techniques]*. Leningrad: Gidrometeoizdat, p. 232. (in Russian)

Dalmatov, B. I. (1958). Usloviia modelirovaniia protsessa pucheniia vodonasyshchennogo grunta [Simulation environment of the heave process of saturated soils]. In: *Proceedings of the Leningrad Institute of Engineering and Construction*. Leningrad, 29, pp. 127–133. (in Russian)

Dalmatov, B. I. (1988). *Mekhanika gruntov, osnovaniia i fundamenty (vkliuchaia spetsial'nyi kurs inzhenernoi geologii) [Mechanics of soils, bases and foundations (including the course on engineering geology)].* Leningrad: Stroyizdat, p. 415. (in Russian)

Dalmatov, B. I. (1954). *Issledovaniia kasatel'nykh sil pucheniia i vliianiia ikh na fundamenty sooruzhenii [Investigation of tangential frost heave forces and their effect on foundations]*. Academy of Sciences of the USSR: Institute of Frost Science, p. 60. (in Russian)

Ershov, E. D. (1985). *Deformatsii i napriazheniia v promerzaiushchikh i ottaivaiushchikh porodakh [Strains and stresses in freezing and thawing rocks].* Moscow: Publishing House of the Moscow University, p. 167. (in Russian)

Ershov, E. D. (1986). *Polevye metody geokriologicheskikh issledovanii [Field methods of geocryological researches].*Moscow: Publishing House of the Moscow University, p. 143. (in Russian)

Frankenstein, S., Tuthill, A. M. (2002). Ice adhesion to locks and dams: past work; future directions? *Journal of Cold Regions Engineering*, 16 (2). DOI: 10.1061/(ASCE)0887-381X(2002)16:2(83)

Golli, O. R. (2000). *Integral'nye zakonomernosti moroznogo pucheniya gruntov i ikh ispol'zovanie pri reshenii inzhenernykh zadach v stroitel'stve [Integral mechanism of frost heaving of soils and application of them in the performance of the engineering challenges in building]*. DSc Thesis in Engineering. All-Russian Vedeneev Hydraulic Engineering Research Institute, p. 45. (in Russian)

Harlan, R. L. (1973). Analysis of coupled heat-fluid transport in partially frozen soil. *Water Resources Research*, 9 (5), pp. 1314–1323. DOI: 10.1029/WR009i005p01314

Hiroshi, S. (2011). Mechanical properties between ice and various materials used in hydraulic structures: The Jin S. Chung Award Lecture, 2010. *International Journal for Offshore and Polar Engineering*, 21(2), pp. 81–90.

Hyang, Y.–H., Cai, Z.–Y., Zhang, C. (2015). Development of centrifugal model test facility for frost heave of channels. *Chinese Journal of Geotechnical Engineering*, 37(4), pp. 615–621. DOI: 10.11779/CJGE201504006

Jiang, H., Tian, Y. (2015). Test for frost heaving damage mechanism of rigid – soften composite trapezoidal canal in seasonally frozen ground region. *Transactions of the Chinese Society of Agricultural Engineering*, 31(16), pp. 145–151.

Karlov, V. D. (1998). *Sezonno promerzayushchie grunty kak osnovaniya sooruzheniy [Seasonal freezing soils as foundations of constructions]*. DSc Thesis in Engineering. Saint Petersburg State University of Architecture and Civil Engineering, p. 349. (in Russian).

Kibriya, T., Tahir, L. (2015). Adfreeze forces on lightly loaded pile foundations of solar PV farms in cold regions. A*merican Journal of Civil Engineering and Architecture*, 3(4), pp. 109–117. DOI: 10.12691/ajcea-3-4-1

Kiselev, M. F. (1971). *Meropriiatiia protiv deformatsii zdanii i sooruzhenii ot deistviia sil moroznogo vypuchivaniia fundamentov [Protective measures against deformations buildings and constructions due to the frost uplift of foundations].* Moscow: Stroyizdat, p. 104. (in Russian)

Konrad, J.- M. (1980). *Frost heave mechanics.* PhD Thesis. University of Alberta, Edmonton, p. 472.

Konrad, J.- M., Morgenstern, N. R. (1980). A mechanistic theory of ice lens formation in fine-grained soils. *Canadian Geotechnical Journal*, 17(4), pp. 473–486. DOI: 10.1139/t80-056

Ladanyi, D., Foriero, A. (1998). Evolution of frost heaving stresses acting on a pile. In: *Proceedings of Seventh International Conference Permafrost*. Yellowknife (Canada), Collection Nordicana, 55, pp. 623–633.

Marov, E. A. (1974). Opredelenie normal'nykh i kasatel'nykh sil moroznogo pucheniia v polevykh usloviiakh [Definition of normal and tangential frost heave forces in field conditions]. In: *Proceedings of designing complex foundations and bases and performace of survey works*. Moscow: Fundamentproekt, pp. 40–49. (in Russian)

Ming, F., Li, D.-Q. (2015). Experimental and theoretical investigations on frost heave in porous media. *Mathematical Problems in Engineering*, ID 198986, pp. 1–9. DOI: 10.1155/2015/198986

Modisette, J. P., Modisette, J. L. (2014). *Pipe Line Frost Heave*. Pipeline Simulation Interest Group, 1421, pp. 1–8. Available at: https://www.atmosi.com/media/1419/1421-pipeline-frost-heave-or-the-lack-thereof-atmos_psig.pdf (viewed on: 11.02.2017).

Nevzorov, A. L. (2000). F*undamenty na sezonnopromerzaiushchikh gruntakh [Foundations on seasonal freezing soils].* Moscow: ASV Publishing House, p. 152. (in Russian)

Parmesvaran, V. R. (1981). Adfreeze strength of model piles in ice. *Canadian Geotechnical Journal*, 18(1), pp. 8–16. DOI: 10.1139/t81-002

Penner, E. (1974). Uplift forces on foundations in frost heaving soils. *Canadian Geotechnical Journal*, 11(3), pp. 323–338. DOI: 10.1139/t74-034

Penner, E. (2010). *Frost-heave uplift forces on foundation*. Available at: https://www.researchgate.net/publication/44047087_Frost-heave_uplift_forces_on_foundations (viewed on: 02.06.2017).

Peppin, S. S. L., Style, R. W. (2012). *The physics of frost heave and ice-lens growth.* Available at: http://eprints.maths.ox.ac.uk/1536/1/finalOR35.pdf (viewed on: 02.07.2017).

Puskov, V. I. (1993). *Silovye vozdeistviia moroznogo pucheniia gruntov na fundamenty sooruzhenii i metody ikh rascheta [Force effects of frost heave on building foundations and methods of their calculation].* DSc Thesis in Engineering. Novosibirsk: Siberian State Transport Academy, p. 37. (in Russian)

Roman, L. T., Kotov, P. I. (2016). Modul' deformatsii merzlykh gruntov pri kompressionnykh ispytaniiakh [Modulus of frozen soil deformation under compression tests]. *Osnovaniia, fundamenty i mekhanika gruntov [Soil Mechanics and Foundation Engineering],* 5, pp. 35–40. (in Russian)

Thomas, H., Cleall, P., Li Y. C., Harris, C., Kern-Luetschg, M. (2009). Modelling of cryogenic processes in permafrost and seasonally frozen soils. *Geotechnique*, 59 (3), pp. 173–184. DOI: 10.1680/geot.2009.59.3.173

Tolkachev, N. A. (1964). Opredelenie otnositel'nyh normal'nyh sil moroznogo puchenija gruntov [Definition of relative frost heave normal forces]. In: *Proceedings of scientific works of the Research Institute of foundations and underground constructions (foundation engineering)*. Moscow: Stroyizdat, pp.165–170. (in Russian)

Tretiakova, O. V. (2016). Velichiny normal'nykh napriazhenii moroznogo pucheniia, razvivaiushchikhsia v glinistykh gruntakh [Values of normal frost heave stresses, developing in clay-bearing soils]. *Transport. Transportnye sooruzheniia. Ekologiia [Transport. Transport Facilities. Ecology]*, 1, pp. 125–141. (in Russian)

Tsytovich, N. A. (1973). *Mekhanika merzlykh gruntov [Mechanics of frozen soils]*. Moscow: Vysshaya Shkola, p. 636. (in Russian)

Vyalov, S. S. (1959). *Reologicheskie svoistva merzlykh gruntov [Rheological properties of frozen soils].* Moscow: Publishing House of the Academy of Sciences of the USSR, p. 191. (in Russian)