CALCULATION OF TANGENTIAL FROST HEAVE STRESSES BASED ON PHYSICAL, MECHANICAL AND STRESS-STRAIN BEHAVIOR OF FROZEN SOIL

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Abstract

Development of northern territories opens up new opportunities for social and economic development of the society. However, designing construction facilities in such areas is complicated by special features of the severe climate; frost heave of soils is one of its manifestations. Normal frost heave forces under the foundation base can be neutralized by foundation embedment below the seasonal frost line. In these conditions, tangential heave stresses will act on the lateral surfaces of foundations, causing uneven lifting of structures, which will lead to the violation of the structure integrity. Regulatory and technical literature recommends determination of tangential frost heave stresses on the basis of generalized tabular data or experimentally, which is not always possible.

A relevant technique for calculating tangential stresses is required, which takes into account climatic and hydrogeological conditions of the construction site. In this article, an attempt is made to reveal those aspects of originating and development of tangential frost heave stresses, which would allow carrying out their calculation. To solve this problem, the existing research experience, features of the cryogenic structure, mechanical and stress-strain behavior of frozen soils were analyzed. A relationship of tangential frost heave stresses with specific cohesion and moisture of frozen soil was revealed on the basis of the analysis.

A point of view, according to which tangential frost heave stresses are a result of maximum compressive heave stresses, directed along the normal to the lateral surface of the foundation, was emphasized too; this allowed relating tangential heave stresses with the frozen soil deformation modulus. Based on the above statements, formulas for calculating tangential frost heave stresses were obtained. The calculation method for determining these stresses would provide the possibility of designing cost-effective and safe structures in areas with the severe climate.

Keywords

Wetness, boundary layer, tangential stresses, deformation modulus, frost heave, specific soil cohesion.

Introduction

In cold regions at the seasonal decrease of air temperature, cooling and freezing of the soil take place, which lead to increase in its volume, i.e. frost heave. Upon limitation of the surface expansion of foundations and embedded structures, normal stresses arise in soils that cause development of forces, which are tangential to these surfaces. Up to date, tangential frost heave stresses, acting on embedded structures, are determined either by generalized tables in regulatory documents, or experimentally. However, the latter is not always possible.

A question arises about the technique of calculating stresses for practical designing. Development of the calculation method, first of all, requires a study of the mechanism of originating and development of specified force factors. According to the existing concepts, tangential frost heave stresses develop upon freezing-up and subsequent interaction of the soil with the foundation material. However, understanding of heterogeneity of interacting media, which are the foundation and the soil, requires the development of these ideas and possible introduction of additional provisions in the definition of the stress-strain state of the "foundation–freezing soil" system. Theoretical prerequisites for this study are modern concepts of the cryogenic structure, mechanical and stress-strain behavior of the frozen soil, as well as existing concepts of the tangential stresses development staging.
The loading factor is an increase in soil volumes at freezing, leading to the formation of a stressed zone around the foundation. The effect of impact of the soil, expanding during freezing in this zone, are normal stresses, which, in their turn, cause stresses and shear forces directed tangentially to the foundation lateral surface.

The following tasks were set: to determine in which layer of the stressed zone shear stresses develop, leading to the development of tangential forces; this calculated layer will determine the qualitative and quantitative picture of the interaction; to reveal properties of interacting media, which would allow calculating tangential stresses; to establish a relationship of the stress value with mechanical and stress-strain properties of media. This would allow obtaining a technique for calculating stresses.

Background

In northern regions, dangerous natural process of frost heave of soils is one of manifestations of the severe climate. Normal stresses and stresses, tangential to foundation surfaces, develop under the influence of frost heave. Tangential stresses cause uneven lifting of structures, leading to their integrity violation and failures of buildings and infrastructure facilities. The influence of tangential stresses on structures is noted in works of B. I. Dalmatov (1954), V. D. Karlov (1998), M. F. Kiselev (1971), A. L. Nevzorov (2000), R. Sh. Abzhalimov (2006), D. Ladanyi (1998), J. P. Modisette (2014), and T. Kibriya (2015). Consideration of frost heave effect is a required condition for designing safe buildings and structures. Analysis of technical literature (https://ohranatruda.ru/ot_biblio/normativ/data_normativ/46/46329/) has shown that at the present time there is no generally accepted technique in regulatory documents for calculating tangential frost heave stresses, which is the reason to use recommended tabular values in practical designing or determine stresses experimentally. However, tabular data do not allow taking into account the entire range of soil and climatic conditions, as well as their possible combinations. These data were obtained by results of experiments, which were carried out for a limited number of soils. Experimental determination of tangential heave stresses on a particular site is not always possible due to limited terms of execution of project documentation and amounts of financing. Experimental estimation, which provides for full-scale or laboratory researches, extends designing terms and implies for an additional item of expenditures for surveying and designing works. A full-scale experiment requires a term, corresponding to the period of negative temperatures of the atmospheric air, tools for measuring tangential heave stresses and special trained technical personnel. Methods of field measuring of tangential stresses are represented in works of B. I. Dalmatov (1954), V. D. Karlov (1998) N. A. Tolkachev (1964), E. A. Marov (1974), E. D. Ershov (1986), A. G. Alekseev (2006), H. Jiang (2015). A laboratory experiment is impossible without proper premises, special certified equipment and scientific and technical maintenance. Methods of the laboratory determination of frost heave forces are shown in the Guidelines (1973). Researches on frost heave on models in laboratory conditions were conducted by E. D. Ershov (1985), V. R. Parmesvaran (1981), Thomas H. (2009), Y.-H. Hyang (2015), and F. Ming (2015).

A question regarding effectiveness of the experimental approach, more inherent to scientific research, to prediction of tangential frost heave stresses during practical designing arises. The use of the approved method for calculating tangential frost heave stresses, based on fundamental studies of frost heave, would allow designing safe structures.

In Russia, the basic theory of frost heave, including the description of normal and tangential stresses, was developed by N. A. Tsytovich (1973), B. I. Dalmatov (1988), and S. S. Vyalov (1959). Abroad, the physics of frost heave is presented in works of R. L. Harlan (1973), J.-M. Konrad (1980), and S. L. Peppin (2012). E. Penner (1974, 2010), S. Frankenstein (2002), K. W. Biggar (2011), and S. Hiroshi (2011) were engaged in determining values of tangential stresses and frost heave forces. According to the existing concepts, the formation of tangential stresses is determined by forces of soil adfreezing with the lateral surface of the foundation. Heave forces, which develop in the surrounding soil mass, tend to move the foundation upwards. At that, frozen soil shear relative to the foundation occurs. Static bonds of the soil adfreezing with the foundation (adfreezing strength) are violated. Dynamic bonds arise which are determined by the resistance to the displacement of the frozen soil layer relative to the lateral surface of the foundation — tangential heave forces.

According to the existing studies, the value of tangential frost heave forces is close to values of steady forces of soil adfreezing with the foundation material. This provision was represented in works by B. I. Dalmatov (1988), N. A. Tsytovich (1973) and S. S. Vyalov (1959). Adfreezing forces are understood as the resistance to frozen soil shear throughout the lateral surface of the foundation. The force of soil adfreezing with the foundation corresponds to the general force which is needed to be applied to it in order to break the bond with the soil frozen around. Thus, it is considered that displacement of the frozen soil relative to the foundation takes place between the foundation material and the soil.

The resistance to frozen soil shear is a function of main variables, which are physical, thermal physical and mechanical properties of the soil, characteristics of the foundation material. Properties of frozen soil are conditioned by its complex cryogenic structure, which, in its turn, is determined by the temperature, freezing rate, external load value and time of its action. Characteristics of the foundation material are related to the characteristics of roughness of its surface. The listed factors indicate heterogeneity of media interacting during shear. Whereas, for shear occurrence directly between the material and frozen soil, these two media shall be homogeneous. All this creates prerequisites for the development of existing provisions on the determination of tangential frost heave stresses.
Theoretical prerequisites of the study

The analysis of the existing works devoted to cryogenic structures, mechanical properties and stress-behavior of frozen soil was carried out. As N. A. Tsytovich (1973) points out, frozen soil represents a four-component system consisting of soil particles, inclusions of ice, unfrozen film water and gaseous inclusions. The amount and phase states of components in the combination with natural properties of the soil determine its heterogeneous cryogenic structure. In this structure, soil particles are "aggregated" by ice-cement. The contact surfaces of particles of the soil and ice form numerous shear planes, which provide the possibility of frozen soil brittle fracture from shear stresses during shear.

Studies of the values of tangential frost heave stresses were carried out by B. I. Dalmatov, Y. D. Dubnov, N. A. Tsytovich, S. S. Vyalov and others. B. I. Dalmatov found a linear dependence of steady adfreezing resistance on temperature (Figure 1), and, based on this, a formula for specific tangential heave force was obtained:

$$\tau_{\text{heave}} = \frac{h_{\text{heave}}}{h} \left( c + b \cdot t_{\text{mean}} \right)$$

where $c$ and $b$ are parameters of the straight line, determined in laboratory conditions and depending on the soil type and its moisture, $c = 4–5 \text{ t/m}^2$; $b = 1–1.5 \text{ t/}^\circ \text{C} \cdot \text{m}^2$; $h_{\text{heave}}$ is thickness of the layer subjected to heave; $h$ is depth of freezing, m; $t_{\text{mean}}$ is mean temperature of the soil within the layer $h_{\text{heave}}$, by the time of its freezing completion.

The non-uniformity of tangential stresses development over time is shown by N. A. Tsytovich (1973), S. S. Vyalov (1959), B. I. Dalmatov (1988), and V. I. Puskov (1993).

Based on the current concepts of staging of the tangential stresses development, the author considered the work of interacting media. At the first stage, shear with soil mass isolation occurs relative to the thin boundary layer of this soil, located on the lateral surface of a pile and frozen together with it. In this case, tangential shear stresses have a significant value, but a small period of activity. These stresses can be characterized as a short-term, almost instantaneous shear occurring when the soil reaches the ultimate resistance to shear.

In this case, ultimate shear strength of particles of the frozen soil boundary layer connected with the surface of the embedded structure, which is practically its "part", is of the paramount importance. Generation of tangential shear stresses is conditioned by overcoming total cohesion of frozen soil particles, corresponding to the ultimate resistance of the soil and ice to shear; normal pressure of the freezing soil on the lateral surface of the foundation.

The activity period of ultimate (instantaneous) tangential shear stresses $\tau_{\text{inst}}$, causing shear with the isolation of soil aggregates in the boundary layer is replaced by the period of decay (stress relaxation), which continues up to the stabilization of the constant soil shear rate throughout the lateral surface of a pile at stresses, which reached the value of $\tau_{\text{steady}}$. Further, the second stage of steady-state tangential stresses $\tau_{\text{steady}}$ begins; these stresses appear when the frozen soil massif is uniformly displaced relative to the boundary layer. The massif displacement occurs due to micro-displacements in the boundary layer, in which a part of bonds (least strong) is already broken at the stage of short-term shear. Here tangential stresses have less importance, but they act during almost the entire freezing period and determine the actual picture of the development of tangential frost heave stresses throughout the lateral surface of the pile. This is confirmed by researches of B. I. Dalmatov (1958), N. A. Tsytovich (1973) and S. S. Vyalov (1959). A diagram of the dependence of shear resistance on time by B. I. Dalmatov is shown in Figure 1b.

At the second stage, relaxation of stresses from $\tau_{\text{inst}}$ to $\tau_{\text{steady}}$ occurs, mainly due to the reduction of frozen soil cohesion forces, under the action of the continuous load from frost heave forces. This phenomenon was described in experiments of S. S. Vyalov (1959).

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**Figure 1.** a) graph of dependence of shear resistance on time; b) graph of dependence of steady adfreezing resistance on soil temperature (B. I. Dalmatov, Y. D. Dubnov).
Characteristic change in cohesion of frozen soils over time, which forms the prevailing part of their shear resistance, is also illustrated in dependences of N. A. Tsytovich (1963). The studies, carried out by S. S. Vyalov (1959) and N. A. Tsytovich (1973), showed that cohesion forces (the main strength characteristic of frozen soil), decreased under the action of shear stresses over time, as a rule, within 1/3 – 1/5 $c_{sat}$. Also, the internal friction angle decreases for clays at continuous shear.

**Methods of calculation of tangential frost heave stresses**

Special features of the cryogenic structure and mechanical properties of frozen soils allowed assuming that shear stresses caused by soil heave occur not along the “foundation–freezing soil” boundary, but in the boundary freezing layer adjacent to the foundation. This proposition is substantiated by aspects considered above, which are the theoretical prerequisites of this study: heterogeneity of interacting continuous media — the foundation material and freezing soil; and the ability of frozen soil for brittle fracturing.

The effect of constrained heave impact near the lateral surface of the foundation is represented by significant compressive forces, leading to the development of shear stresses, which are realized throughout numerous shear planes formed due to different shear strength of soil components in the calculated boundary layer. Mechanical properties of ice are analyzed in the work of V. V. Bogorodskiy (1983). Tangential forces are the resultant factor of these shear stresses. Violation of bonds between particles of the soil and ice is needed for origination of shear deformations. Shear in any elemental fragment will take place between ice layers or along the boundary of ice and mineral interlayers due to the lower shear strength of soil particles. In some approximation, it is possible to estimate the distribution of shear deformations according to the percent ratio between the soil and ice. This allowed establishing a relationship of the value of tangential stresses to the percent ratio between the soil and ice. This instantaneous stress is the ultimate shear force of the frozen soil layer by the time of its freezing completion.

As B. I. Dalmatov points out, shear stresses originating in cohesive clay-bearing soils near the lateral surface of the foundation, are the following:

$$\tau = c + \sigma \cdot t \cdot \varphi$$

where $c$ is specific cohesion; $\sigma$ is normal pressure; $\varphi$ is internal friction angle.

It is assumed that the dependence (3) can be used to determine tangential shear stresses of frost heave, developing on the lateral surface of foundations, using basic variables of B. I. Dalmatov’s formula for the specific tangential frost heave force. The B. I. Dalmatov’s formula for the specific tangential frost heave force has the following form:

$$\tau_{max} = \frac{h_{heave}}{h}(c + b \cdot t_{mean})$$

where $c$ and $b$ are parameters of the straight line, determined in laboratory conditions and depending on the soil type and its moisture, $c = 4 – 5 \ t/m^2$; $b = 1 – 1.5\ t/0°C\cdot m^2$; $h_{heave}$ is thickness of the layer subjected to heave; $h$ is depth of freezing, $m$; $t_{mean}$ is mean temperature of the soil within the layer heave by the time of its freezing completion.

Main variables are mechanical properties of the soil, moisture, temperature of the frozen soil, thickness of the freezing soil layer by the time of its freezing completion.

As per the above, tangential stresses at the first stage can be expressed as follows according to equation (3):

$$\tau_{inst} = c_{inst} + p_{mean} \cdot t \cdot \varphi_i$$

where $c_{inst}$ is total (instantaneous) cohesion of frozen soil particles, corresponding to the ultimate resistance, kPa; $p_{mean}$ is mean pressure on the lateral surface of the foundation contacting with the frozen soil, kPa; $\varphi_i$ is angle of internal friction between the soil and ice.

The mean pressure on the lateral surface of the foundation has the following form:

$$P_{mean} = \frac{\xi_i \cdot \gamma_i \cdot z_i}{1 + v_i}$$

where $\xi_i$ is coefficient of the lateral pressure of the soil and ice.

$$\xi_i = \frac{\gamma_i}{1 - v_i}$$

$\gamma_i$ is volume density of the soil and ice, kPa; $z_i$ is depth (thickness) of $i$th layer of the frozen soil, m; $v_i$ is Poisson’s ratio for the soil and ice.

Substituting formulas (2) and (5) into formula (4), we obtain the maximum tangential frost heave stress within the boundary layer at the depth $z_i$ at the specified soil moisture. This instantaneous stress is the ultimate shear strength of the frozen soil.
\[ \tau_{\text{inst}} = \tau_{\text{inst}} = \left[ \left( c_{\text{soil}} \cdot z \cdot \left( 1 - 1.09 \cdot \rho_{w} \right) \right) + \left( 1 - 1.09 \cdot \rho_{w} \right) \right] + \zeta_{\text{soil}} \cdot \gamma_{c} \cdot z^2 \cdot \rho_{\text{sat}} \cdot 1.09 \cdot \rho_{w} \]  

At the second stage, steady-state tangential stresses develop, which are more reliable indices than the short-term ones at the first stage. Steady-state stresses are characterized by a decrease in shear stresses due to a decrease in the specific cohesion of the soil and angle of internal friction for cohesive soils, according to N. A. Tsytovich (1973) and S. S. Vyalov (1959). Based on this, it is possible to determine the steady-state stresses from equation (6), replacing the instantaneous cohesion \( c_{\text{inst}} \) of the soil by its continuous value \( c_{\text{con}} \):

\[ \tau_{\text{stead}} = \left[ c_{\text{soil}} + c_{\text{ice}} \right] + \zeta_{\text{soil}} \cdot \gamma_{c} \cdot z^2 \cdot \rho_{\text{sat}} \cdot 1.09 \cdot \rho_{w} \]  

Analyzing the moisture value \( w \) in equation (7), let us note that the ice formation in the freezing soil is conditioned by the total moisture content, including the pore moisture in the natural composition of the soil and the moisture entering as a result of migration:

\[ w_{I} = w + w_{\text{migr}} \]  

Let us substitute the total moisture content (8) instead of \( w \) into equation (7):

\[ \tau_{\text{stead}} = \left[ c_{\text{soil}} - 1.09 \cdot (w + w_{\text{migr}}) \right] + \zeta_{\text{soil}} \cdot \gamma_{c} \cdot z^2 \cdot \rho_{\text{sat}} \cdot 1.09 \cdot (w + w_{\text{migr}}) \]  

Let us determine the share of the migration moisture in the freezing soil, using the formula for heave deformation proposed by A. L. Nevzorov (2000). In this equation, the second summand is connected with the moisture entering the freezing zone due to migration forces:

\[ h_{\text{migr}} = 0.09 \cdot (w_{\text{sat}} - w_{\text{inst}}) \cdot \frac{\rho_{w}}{\rho_{v}} \cdot z + 1.09 \cdot S \cdot \tau \cdot \text{grad} t \]  

Then, the moisture forming due to water migration to the cold front, according to equation (10), will be:

\[ w_{\text{migr}} = \frac{S \cdot \tau \cdot \text{grad} t}{z} \]  

After substitution of (11), equation (9) will take the following form:

\[ \tau_{\text{stead}} = \left[ c_{\text{soil}} - 1.09 \cdot (w + S \cdot \tau \cdot \text{grad} t) \right] + \zeta_{\text{soil}} \cdot \gamma_{c} \cdot z^2 \cdot \rho_{\text{sat}} \cdot 1.09 \cdot (w + S \cdot \tau \cdot \text{grad} t) \]  

As it was noted above, main variables of the B. I. Dalmatov's formula for specific tangential heave strength are the following: mechanical properties of the frozen soil, moisture, temperature of the frozen soil by the time of its freezing completion, thickness of the freezing layer. As it was mentioned above, main variables of equation (13) are the following: mechanical properties of the frozen soil, moisture, temperature gradient within the frozen layer; freezing time, segregated soil potential (characterized by heave velocity, moisture migration rate and temperature gradient), and thickness of the freezing layer.

The reason of the generation of tangential stresses on the lateral surface of the foundation is maximum normal compressive stresses developing when limiting expansion of the heaving soil during its freezing. This proposition was used in obtaining the method for calculating tangential frost heave stresses. The relationship of tangential stresses with normal frost heave stresses was found by V. I. Puskov (1973). V. I. Puskov notes that "in the absence of a local load, on the soil surface in the layer of seasonal freezing, volume deformation \( f \) is completely realized in the direction of the vertical axis and includes, besides \( f_{z} \), lateral heave deformations transformed in the direction of the axis by the arising lateral pressure \( \sigma_{l} \)."
Stating methods for determining heave deformations, V. D. Karlov (1998) in his DSc thesis noted that “the considered methods also allow determining the value of specific forces of soil frost heave depending on the degree of constraint of heave deformations”. And the degree of constraint of heave deformations, in its turn, conditions normal frost heave stresses, which can be traced in works of various authors. For example, according to N. A. Tsytochuk (1973), “the order of the maximum value of normal frost heave forces can be estimated based on the values of pressures which ice crystals develop upon the constrained water freezing”. According to E. D. Ershov, “stresses in freezing rocks are conditioned by the exclusion of heave deformations”. Thus, the existing studies allowed expressing tangential frost heave stresses with the use of the Coulomb law in dependence to horizontal normal frost heave stresses $\sigma_x$:

$$\tau = \sigma_x \cdot t \cdot g \cdot \phi$$  \hspace{1cm} (14)

where $\sigma_x$ is frost heave stress normal to the lateral surface of the foundation according to the formula obtained by the author (2016); $\phi$ is angle of internal friction of the frozen soil or ice.

The author obtained the formula for calculating horizontal frost heave stresses $\sigma_x$, considering properties of the frozen soil, depth (thickness) of the freezing layer and freezing conditions:

$$\sigma_x = \sigma_{\text{w}} \cdot 1.09 \cdot (S \cdot P \cdot \text{grad} \cdot \frac{E_m}{z} \cdot \left[1 - e \left(1 - w_c \cdot \frac{\rho_w}{\rho_c} - 0.9 \cdot w_c \cdot \frac{\rho_d}{\rho_w} \right)\right]) \cdot k_{\text{aw}}$$  \hspace{1cm} (15)

where $E_m$ is modulus of frozen soil deformation; $w$ is natural moisture of the soil; $w_c$ is moisture by unfrozen water; $e$ is porosity factor of the soil; $z$ is depth of soil freezing in the vertical direction; thickness of the soil layer in the horizontal direction; $SP$ is segregated soil potential; $t$ is freezing time; $\text{grad}$ is temperature gradient; $\rho_p$ is density of the dry soil; $\rho_c$ is density of free water; $\rho_d / \rho_w$ is a factor of conversion of the mass moisture into the volume moisture; $k_{\text{aw}}$ is an anisotropy factor of the frozen soil, taking into account the direction of freezing and frost heave stresses.

Then equation (14) for tangential frost heave stresses will take the following form:

$$\tau = \frac{1.09 \cdot (S \cdot P \cdot \text{grad} \cdot \frac{E_m}{z} \cdot \left[1 - e \left(1 - w_c \cdot \frac{\rho_w}{\rho_c} - 0.9 \cdot w_c \cdot \frac{\rho_d}{\rho_w} \right)\right]) \cdot k_{\text{aw}}}{\phi \cdot \rho c}$$  \hspace{1cm} (16)

Due to the unfrozen water, the soil moisture is determined as a function of the plastic limit moisture (in unit fractions), according to Regulations SP 25.13330.2012:

$$w_c = K_w \cdot w_p$$

where $k_w$ is a factor taken from table B.3 [Regulations SP 25.13330.2012] in dependence to the index of plasticity and soil temperature; $w_p$ is soil moisture at the plastic limit.  

Kujala found that the segregated potential $SP$ (mm3/h·°C) was in correlation with the heave velocity $h_v$ (mm/day):

$$S \cdot P = 1.1 \cdot v_h = m \cdot m^2 / \text{hour} \cdot \circ^\circ C$$

where $v_h$ is heave velocity, mm/day. The mean temperature gradient for the layer $z$ is calculated as the ratio of the mean winter temperature to 0.5 $z$:

$$\text{grad} = \frac{t_{h,f} - t_z}{0.5 \cdot z} = \frac{t_z}{0.5 \cdot z} \cdot \circ^\circ C / m$$

where $t_{h,f}$ is temperature at the beginning of freezing $t_{h,f} \rightarrow 0$; $t_z$ is mean winter temperature; $z$ is thickness of the freezing edge (freezing layer).

The time of freezing amounts to:

$$t = \frac{z}{v_n}$$

where $v_n$ is soil freezing velocity.

Equation (15) after the substitution of values will take the following form:

$$\sigma_x = 1.09 \cdot (1.1 \cdot v_h) \cdot \frac{z}{v_n} \cdot \frac{t}{0.5z} \cdot \frac{E_m}{z} \cdot \left[1 - e \left(1 - w_c \cdot \frac{\rho_w}{\rho_c} - 0.9 \cdot w_c \cdot \frac{\rho_d}{\rho_w} \right)\right] \cdot k_{aw}$$  \hspace{1cm} (17)

After transformations we get the following:

$$\sigma_x = 2.4 (m^2/\circ^\circ C) \cdot \frac{v_h}{v_n} \cdot \frac{t}{z} \cdot \frac{E_m}{z} \cdot \left[1 - e \left(1 - w_c \cdot \frac{\rho_w}{\rho_c} - 0.9 \cdot w_c \cdot \frac{\rho_d}{\rho_w} \right)\right] \cdot k_{aw}$$  \hspace{1cm} (18)

Then, equation (16) will take the following form:

$$\tau = 2.4 (m^2/\circ^\circ C) \cdot \frac{v_h}{v_n} \cdot \frac{t}{z} \cdot \frac{E_m}{z} \cdot \left[1 - e \left(1 - w_c \cdot \frac{\rho_w}{\rho_c} - 0.9 \cdot w_c \cdot \frac{\rho_d}{\rho_w} \right)\right] \cdot k_{aw} \cdot t \cdot g \cdot \phi$$  \hspace{1cm} (19)

It is rather difficult to determine the modulus of frozen soil deformation. In regulatory documents, it is recommended to determine this parameter experimentally. According to the recommendations of “Guidelines for Determining Physical, Thermal Physical and Mechanical Properties of Frozen Soils” (https://ohranatruda.ru/ot_biblio/normativ/data_normativ/48/480089/), the deformation modulus is the value reciprocal to the compressibility. The compressibility, in its turn, is determined depending on the volume weight $\rho$ and soil moisture $w$.

Then the following basic physical values can be taken as parameters determining the value of tangential stresses in equation (19):

$$f(w, t, z, v_h, \rho, t \cdot g \cdot \phi) = 0$$  \hspace{1cm} (20)
The reason of the generation of tangential stresses is compressive heave stresses, normal to the lateral surface of the foundation and leading to shear. Formula (16) for tangential stresses calculation is proposed, as a function of the specified normal pressure. The latter was obtained by the author (2016) on the basis of physical properties and strain behavior of the frozen soil, freezing conditions, and regularities of moisture migration.

The values calculated according to formulas (13) and (16) do not contradict the data of other researchers.

Table 2. Comparison of tangential frost heave stresses

<table>
<thead>
<tr>
<th>Author of the study</th>
<th>Type of the study</th>
<th>Freezing depth, m</th>
<th>Tangential heave stress, kN/m²</th>
<th>Mean tangential heave stress, kN/m²</th>
</tr>
</thead>
<tbody>
<tr>
<td>O. V. Tretiakova (2017)</td>
<td>Calculation</td>
<td>1.0</td>
<td>79–152.7</td>
<td>115.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0–0.5</td>
<td>270–300</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>0–1.0</td>
<td>140–160</td>
<td></td>
</tr>
<tr>
<td>R. Sh. Abzhalimov (2006)</td>
<td>Field study</td>
<td>120</td>
<td>270–300</td>
<td></td>
</tr>
<tr>
<td>D. Ladanyi, A. Foriero Figure 2 (1998)</td>
<td>Field study</td>
<td>1.12</td>
<td>60–160</td>
<td>120</td>
</tr>
</tbody>
</table>

Discussion

Currently, issues of the uneven distribution of tangential stresses throughout the depth and variability of their values over time remain controversial. Studies of the frozen soil deformation moduli will help to take into account the non-uniformity of tangential stresses throughout the depth. These indices can be determined in laboratory conditions. In Russia, strength properties and strain behavior of frozen soils were studied by L. T. Roman (2016), R. Sh. Abzhalimov (2009, 2011) and others. Abroad, these issues were analyzed in the works of T. F. Azmatch (2011), K. W. Biggar (2011) and others. The carried out studies are useful for the increase of applicability of the calculating formulas obtained by the author.

An important role in the development of calculation techniques belongs to rheological properties of the soil, which determine the unevenness of tangential stresses over time.

Long-term soil strength is more relevant for assessing the suitability of the construction safety for the entire lifetime than the short-term breaking ultimate strength limit obtained during tests. This area represents a field for further researches.
References

Abzhalimov, R.Sh. (2006). Opredelenie velichiny vypuchivaniia grunta vokrug svi, samikh svi i svainykh rostverkov, usilii v nikha puchinistykh gruntakh [Determination of soil uplift gradient around piles and pile grids, their stresses on heaving soils]. International Journal. Razvitie gorodov i gidrotekhnicheskoe stroitel'stvo [City development and hydraulic engineering], 10, pp. 214–221. (in Russian)


Abzhalimov, R.Sh. (2011). Zakonomernosti vzaimodeistviia puchinistogo gruntovogo osnovaniia s fundamentami maloetazhnykh zdаний i podzemnymi sooruzheniami i metody ikh rascheta [Regularities of the interaction of heaving soil foundations with foundations of low-storey buildings and underground structures and methods of their calculation]. DSc Thesis in Engineering. Moscow: Moscow State University of Railway Engineering, p. 47. (in Russian)


