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PROBLEM OF THE ANISOTROPY OF ELASTICITY AND STRENGTH IN ANISOTROPIC FIBER MATERIALS

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Abstract
Introduction: The paper presents new results of studies on the anisotropy of fiber materials with cylindrical anisotropy, which include filament-wound composite materials reinforced with various fibers. **Methods:** We suggest a mathematical solution to a fourth-order partial differential equation in polar coordinates with two variables for an orthotropic anisotropic body. To solve this equation, we converted it into Cartesian coordinates and presented the stress function as a sum of polynomials. **Results and Discussion:** As a result of the solution, we obtained two relationships between the elastic constants in the principal directions of anisotropy (so-called elasticity parameters). One of them was obtained for the first time, and the other results from the solution of the anisotropy problem for an orthotropic curved body, suggested by S. G. Lekhnitsky. The obtained solution does not contradict Lekhnitsky's solution. Thus, in our opinion, orthotropic materials can be divided into two groups. In one group, when shifting from the radial to the tangential direction, the elastic constants take on extreme values when the layers are at angles of 0, 60, and 90°. In the other group, there is no intermediate extreme value and the elastic constants take on extreme values when the layers are at angles of 0 and 90°. The obtained results can be applied in the development of new high-strength composite materials and new technologies for the design and manufacture of building structures, as well as in the design of high-strength structures from synthetic composite materials.

Keywords
Anisotropy of properties, composite materials, mathematical model, cylindrically anisotropic body, modulus of elasticity, principal stresses, elastic constants, Poisson's ratio, shear modulus.

Introduction

Filament-wound composites reinforced with fibers of carbon, boron or basalt, metallic or glass fibers, and wood as a natural composite material can be classified as anisotropic fiber materials with cylindrical anisotropy.

In nature, composite materials are formed in a natural way, and, according to the basic principles of bionics, their strongest fibers extend in the direction of principal stresses and strains.

By studying natural composite materials, material engineers design materials with pre-determined properties.

Fiber-glass products (including those made of filament-wound fiber-glass), similar to such a natural composite as wood with its annual rings, get widespread use. Wood, as well as fibers in bones of humans and animals, are often taken as prototypes when creating new advanced high-strength materials.

Such materials are anisotropic. Their physical and mechanical properties vary throughout the volume and in different directions, depending on the required performance of the material.

Many researchers in Russia (Ye. K. Ashkenazi, A. I. Kuznetsov, S. G. Lekhnitsky, A. N. Mitinsky, A. A. Pozdnyakov, A. L. Rabinovich, Yu. S. Sobolev, and others) and abroad (C. S. Grove, A. Jlinen, R. Keylwerth, H. Kubler, D. V. Rosato, and others) have been studying the anisotropy of elasticity and strength in anisotropic materials.

Based on the analysis of literature sources, we established that until recently there was no mathematical relationship found between the elastic constants in the principal directions of anisotropy (in contrast with isotropic bodies). According to some researchers (Ye. K. Ashkenazi, Yu. S. Sobolev, and others), this is one of the main reasons for obtaining conflicting experimental and theoretical results. This prompted new studies on elasticity and strength in anisotropic materials and, in particular, composite materials of natural and synthetic origin.

Over the last 15–20 years, new promising data were obtained for anisotropic materials. In most cases, the insights of 50–70 years ago are still used when conducting research or solving practical problems.

In this paper, we present the results of our theoretical studies on the anisotropy of elasticity in filament-wound composite materials, based on the well-known laws of mathematics and mechanics, and compare those with the results obtained much earlier by Ye. K. Ashkenazi, S. G. Lekhnitsky, Yu. S. Sobolev, and others.

Methods

The following fourth-order homogeneous partial differential equation in polar coordinates for an orthotropic body, known in the theory of elasticity of an anisotropic body (Ashkenazi, 1978; Lekhnitsky, 1977) was taken as the basis for theoretical studies:

$$\begin{gathered}\frac{1}{E_t}\cdot\frac{\partial^4 F}{\partial r^4}+\left(\frac{1}{G_{rt}}-\frac{2\nu_{rt}}{E_r}\right)\cdot\frac{1}{r^2}\cdot\frac{\partial^4 F}{\partial r^2 d\theta^2}+\frac{1}{E_r}\cdot\frac{1}{r^4}\cdot\frac{\partial^4 F}{d\theta^4}+\\ \frac{2}{E_t}\cdot\frac{1}{r}\left(\frac{\partial^3 F}{\partial r^3}\right)-\left(\frac{1}{G_{rt}}-\frac{2\nu_{rt}}{E_r}\right)\cdot\frac{1}{r^3}\cdot\frac{\partial^3 F}{\partial r d\theta^2}- \qquad (1)\\ -\frac{1}{E_r}\cdot\frac{1}{r^2}\cdot\frac{\partial^2 F}{\partial r^2}+\left(2\frac{1-\nu_{rt}}{E_r}+\frac{1}{E_{rt}}\right)\frac{1}{r_4}\cdot\frac{\partial^2 F}{d\theta^2}+\frac{1}{E_r}\cdot\frac{1}{r^3}\cdot\frac{\partial F}{\partial r}=0,\end{gathered}$$

where E_r, E_t — the moduli of elasticity in tension (compression) in the principal directions;
ν_{rt}, G_{rt} — the Poisson's ratio and shear modulus of elasticity.

To solve the problem, Eq. (1) was converted into Cartesian coordinates, which is too cumbersome and, therefore, is not presented in the paper.

To solve Eq. (1) in the plane problem for a circular plate with cylindrical anisotropy, the stress function was taken as a sum of polynomials (Kurdyumov, 1946):

$$F=\sum_{i=1}^{n}x^k\cdot f_k(y), \qquad (2)$$

where $f_k(y)$ — unknown functions satisfying differential Eq. (1).

Results

As a result of solving Eq. (1) with the substitution of the corresponding derivatives of stress function (2), after transformations, second-order algebraic Eq. (3) was obtained, the roots of which are as follows:

$$B^2-\frac{2}{3}\left(5+k^2\right)B-\frac{5}{3}k^4+\frac{14}{3}k^2+1=0; \qquad (3)$$

$$B_{(1)}=3-k^2; \qquad (4)$$

$$B_{(2)}=\frac{1+5k^2}{3}, \qquad (5)$$

where $k^2=E_t/E_r$.

One of the roots (4) can be found in the monograph by S. G. Lekhnitsky (1957), solving the bending problem in an anisotropic curved bar (an orthotropic bar with cylindrical anisotropy), where Eq. 24.7 on page 98 is written as follows:

$$\frac{E_t}{E_r}\left(1-2\nu_{rt}\right)+\frac{E_t}{G_{rt}}=3. \qquad (6)$$

By substituting the known relationships into Eq. (6), we obtain the same expression (4) for the first root, which does not contradict the relationship between the elastic constants, obtained by S. G. Lekhnitsky. However, we also obtained expression (5), which we could not find in any of the known publications. Thus, we can conveniently divide orthotropic anisotropic materials with cylindrical anisotropy into two groups. In the first group of materials satisfying condition (4), the modulus of elasticity changes from 0 to 90° (from the radial to the tangential direction) passing through an intermediate extremum point when the layers are at an angle of 30° to the force line.

In the second group, there is no intermediate extremum point and the modulus of elasticity changes from 0 to 90° smoothly.

This conclusion can be reached by analyzing the equations we obtained for the elastic constants:

$$\frac{1}{E_{x'}}=\frac{cos^4\theta}{E_r}+\frac{sin^4\theta}{E_t}+\frac{3-k^2}{E_t}sin^2\theta\cdot cos^2\theta;$$

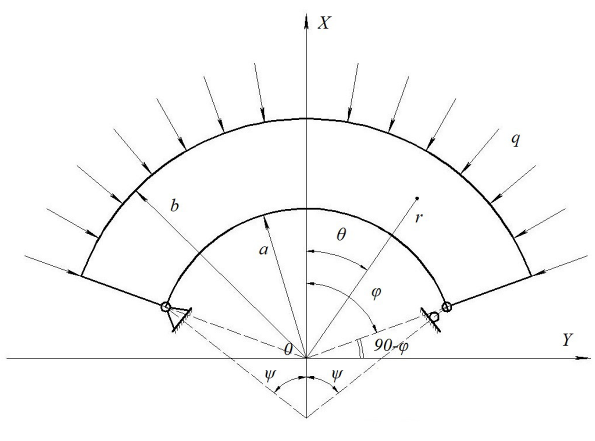
$$\frac{1}{E_{y'}}=\frac{sin^4\theta}{E_r}+\frac{cos^4\theta}{E_t}+\frac{3-k^2}{E_t}sin^2\theta\cdot cos^2\theta;$$

$$\frac{1}{G_{x'y'}}=\frac{8\left(k^2-1\right)}{E_t}sin^2\theta\cdot cos^2\theta+\frac{1}{G_{rt}};$$

$$\frac{1}{\nu_{x'y'}}=-E_{x'}\left[\frac{2\left(k^2-1\right)}{E_t}sin^2\theta\cdot cos^2\theta-\frac{\nu_{rt}}{E_r}\right].$$

According to our solution, the coefficient β, irrespective of $k^2=E_t/E_r$, will be equal to 2 (as in the monograph by S. G. Lekhnitsky (1957)), i.e. it will be exactly the same as in an isotropic bar, and this does not contradict the conclusion drawn by S. G. Lekhnitsky:

$$\beta=\sqrt{1+\frac{a_{11}+2a_{12}+a_{66}}{a_{22}}}=\sqrt{1+\frac{E_t}{E_r}\left(1-2\nu_{rt}\right)+\frac{E_t}{G_{rt}}}.$$



Then the stresses in an orthotropic cylindrically anisotropic curved bar can be calculated using the equations in Lekhnitsky's notations (1957):

$$\left.\begin{aligned}
&\sigma_r=\frac{q}{h}\left[P+Q\left(\frac{r}{b}\right)^{k-1}+R\left(\frac{b}{r}\right)^{k+1}\right]+\\
&+\frac{q}{b\cdot h\cdot g_1}\cdot\frac{b}{r}\left[\left(\frac{r}{b}\right)^{\beta}+C^{\beta}\left(\frac{b}{r}\right)^{\beta}-\left(1+c^{\beta}\right)\right]\frac{\cos(\varphi-\psi)}{cos\psi}\cdot cos\theta,\\
&\sigma_t=\frac{q}{h}\left[P+Q\ k\left(\frac{r}{b}\right)^{k-1}-R\ k\left(\frac{b}{r}\right)^{k+1}\right]+\\
&+\frac{q}{b\cdot h\cdot g_1}\cdot\frac{b}{r}\left[(1+\beta)\left(\frac{r}{b}\right)^{\beta}+(1-\beta)c^{\beta}\left(\frac{b}{r}\right)^{\beta}\right.\\
&\left.-\left(1+c^{\beta}\right)\right]\frac{\cos(\varphi-\psi)}{cos\psi}\cdot cos\theta,\\
&\tau_{rt}=\frac{q}{b\cdot h\cdot g_1}\cdot\frac{b}{r}\left[\left(\frac{r}{b}\right)^{\beta}+c^{\beta}\left(\frac{b}{r}\right)^{\beta}-\left(1+c^{\beta}\right)\right]\frac{\cos(\varphi-\psi)}{cos\psi}\cdot sin\theta,\\
&where\ P=\frac{1}{2\left(k^2-1\right)\left(1-c^{2k}\right)g}\ \mathrm{x}
\end{aligned}\right\}\quad(7)$$

$$\left.\begin{aligned}
&\mathrm{x}\left[\begin{aligned}&2k(k-1)\left(1-c^{k+1}\right)+2k(k+1)c^{k+1}\left(1-c^{k-1}\right)-\\&\left(k^2-1\right)(1+c)\left(1-c^{2k}\right)m\end{aligned}\right];\\
&Q=\frac{1}{2(k-1)\left(1-c^{2k}\right)g}\left[\begin{aligned}&-(k-1)\left(1-c^2\right)-2kC^2\left(1-C^{k-1}\right)\\&+(k-1)(1+C)(1-C)m\end{aligned}\right];\\
&R=\frac{1}{2(k+1)\left(1-c^{2k}\right)g}\mathrm{x};\\
&\mathrm{x}\left[\begin{aligned}&(k+1)c^{2k}\left(1-c^2\right)-2kc^{k+1}\left(1-c^{k+1}\right)-(k+1)\\&(1+c)c^{2k}\left(1-c^{1-k}\right)m\end{aligned}\right];\\
&c=\frac{a}{b};m=\frac{sin\varphi\ \ sin(\varphi-\psi)}{cos\psi},
\end{aligned}\right\}\quad(8)$$

where

$$k=\sqrt{\frac{E_t}{E_r}};g=\frac{1-c^2}{2}-\frac{k}{k+1}\cdot\frac{\left(1-c^{k+1}\right)^2}{1-c^{2k}}+$$

$$\frac{k\cdot c^2}{k-1}\cdot\frac{\left(1-c^{k-1}\right)^2}{1-c^{2k}};$$

$$g_1=\frac{2}{\beta}\left(1-c^{\beta}\right)+\left(1+c^{\beta}\right)lnC.$$

During calculations for the second group of anisotropic materials by Eqs. (7) and (8), we should note that the coefficient β depends on k^2, in contrast with the first case: this follows from Eq. 24.7 (Lekhnitsky, 1957) and the second root of algebraic Eq. (3).

$$B=\frac{1+5k^2}{3},\ \text{i.e.}$$

$$\beta=\sqrt{1+\frac{E_t}{E_r}(1-2\nu_r)+\frac{E_t}{G_{rt}}}=\sqrt{1+k^2+\frac{1+5k^2}{3}}==2\sqrt{\frac{1+2k^2}{3}}.$$

The equations to calculate the elastic constants at the second root will take the following form:

$$\frac{1}{E_{x'}}=\frac{cos^4\theta}{E_r}+\frac{1+5k^2}{3}sin^2\theta\cdot cos^2\theta+\frac{sin^4\theta}{E_t};$$

$$\frac{1}{E_{y'}}=\frac{sin^4\theta}{E_r}+\frac{1+5k^2}{3}sin^2\theta\cdot cos^2\theta+\frac{cos^4\theta}{E_r};$$

$$\frac{1}{G_{x'y'}}=\frac{8\left(1-k^2\right)}{3E_t}sin^2\theta\cdot cos^2\theta+\frac{1}{G_{rt}};$$

$$\frac{1}{\nu_{x'y'}}=-E_{x'}\left[\frac{2\left(1-k^2\right)}{3E_t}sin^2\theta\cdot cos^2\theta-\frac{\nu_{rt}}{E_r}\right].$$

The ratios of the elastic constants (conventionally called the elasticity parameter) depend on the degree of accuracy in determining each of the constants. The Poisson's ratio is rather small. Besides, the authors of methods for its determination (Ye. K. Ashkenazi, A. N. Mitinsky, Yu. S. Sobolev) acknowledge that when the composite layers are at an angle of 45°, there is an inaccuracy in the determination.

The obtained values of the anisotropy parameters make it possible to eliminate this drawback and facilitate the solution of differential Eq. (1).

Therefore, the calculation of stresses for orthotropic anisotropic bodies with cylindrical anisotropy, performed according to Eqs. (7) and (8) with the use of the coefficient β = 2 in one case and $\beta=2\sqrt{\frac{1+2k^2}{3}}$ in the other case, depends on the ratio between the moduli of elasticity $E_t/E_r=k^2$ for a particular anisotropic material.

In this case, it will be necessary to determine in advance which group the anisotropic material belongs to.

Discussion. Extrema of elasticity characteristics

The extremum properties and the position of the principal anisotropy planes are of the most interest when studying the physical and mechanical properties of anisotropic materials, including composite materials of natural and synthetic origin.

The analytical dependence of the modulus of elasticity is known from the theory of elasticity:

$$\frac{1}{E_{x'}}=\frac{cos^4\theta}{E_x}+\frac{sin^4\theta}{E_y}+\left(\frac{4}{E_{xy}^{(45)}}-\frac{1}{E_x}-\frac{1}{E_y}\right)sin^2\theta\cdot cos^2\theta.\quad(9)$$

With the introduction of the following notations:

$$E_x=E_0;E_y=E_{90};E_{xy}^{(45)}=E_{45};E_{x'}=E_{\alpha};$$

$$G_{xy}=G_0;G_{xy}^{(45)}=G_{45};G_{x'y'}=G_{\alpha};\ \mu_{xy}=\mu_0;$$

$$\mu_{yx}=\mu_{90};C=\frac{E_0}{E_{90}};b=\frac{E_0}{E_{45}}-\frac{1+C}{4}.$$

Ye. K. Ashkenazi obtained the following instead of Eq. (9):

$$\frac{E_\alpha}{E_0} = \frac{1}{cos^4\theta + b \cdot sin^2 2\theta + C \cdot sin^2\theta} \ .$$

This equation can be applied to any plane of elastic symmetry.

The equation suggested by A. N. Mitinsky (1948) for an orthotropic body in his notations has the following form:

$$G_{xy} = \frac{E_{xy}^{(45)}}{2\left(1+\mu_{xy}^{(45)}\right)} \ .$$

This equation is similar to the equation for isotropic materials:

$$G_{xy}^{(45)} = G_{45} = \frac{E_0 \cdot E_{90}}{E_0\left(1+\mu_{90}\right) + E_{90}\left(1+\mu_0\right)}.$$

By equating the first derivative $\frac{dE_\alpha}{d\alpha}$ to zero, Ye. K. Ashkenazi obtained the following:

$$\left[cos^2\alpha - Csin^2\alpha - 2b\left(1-2sin^2\alpha\right)\right]sin\alpha \cdot cos\alpha = 0 \ .$$

The first two extrema can be found by equating the factor outside the brackets to zero. Thus, we get the following:

$$\alpha_1 = 0°; \alpha_2 = 90°.$$

The third extremum of the modulus of elasticity will be achieved at the following angle:

$$\alpha_3 = arcsin\sqrt{\frac{1-2b}{1+C-4b}};$$

or with account for substitution 2.31 (Ashkenazi and Ganov, 1981):

$$\alpha_3 = arcsin\sqrt{\frac{1-2\frac{E_0}{E_{45}}+\frac{1+\frac{E_0}{E_{90}}}{2}}{1+\frac{E_0}{E_{90}}-4\frac{E_0}{E_{45}}+\left(1+\frac{E_0}{E_{90}}\right)}}.$$

For an equally reinforced material (at C = 1), e.g. plywood, α_3 = 45°.

The third extremum will be achieved only subject to the following inequality (Ashkenazi and Ganov, 1981):

$$1 \geq \frac{1-2b}{1+C-4b} \geq 0.$$

If this condition is not met, the modulus of elasticity will have only two extrema, as, for instance, in filament-wound fiber-glass or natural wood in the plane of the greatest stiffness (in the direction of the fibers).

The modulus of elasticity *Ex'* of an orthotropic anisotropic material in an arbitrary direction can be determined by expression 2.28 from the monograph by Ye. K. Ashkenazi (1978).

For an arbitrary direction of the *X'* axis, with account for the known ratio between the direction cosines:

$$n_1^2 + l_1^2 + m_1^2 = 1.$$

Ye. K. Ashkenazi obtained the values of the direction cosines:

1) at n_1 = 0;

$$m_1 = \pm\sqrt{\frac{\frac{3}{E_y}-\frac{4}{E_{yz}^{(45)}}+\frac{1}{E_z}}{\frac{4}{E_y}+\frac{4}{E_z}-\frac{8}{E_{yz}^{(45)}}}}; l_1 = \pm\sqrt{(1-m_1^2)};$$

2) at m_1 = 0;

$$n_1 = \pm\sqrt{\frac{\frac{3}{E_y}+\frac{1}{E_x}-\frac{4}{E_{xy}^{(45)}}}{\frac{4}{E_y}+\frac{4}{E_x}-\frac{8}{E_{xy}^{(45)}}}}; l_1 = \pm\sqrt{(1-n_1^2)};$$

3) at l_1 = 0;

$$n_1 = \pm\sqrt{\frac{\frac{4}{E_z}-2A}{\frac{4}{E_x}+\frac{4}{E_z}-4A}}; m_1 = \pm\sqrt{1-n_1^2},$$

where $A = \frac{4}{E_{xz}^{(45)}} - \frac{1}{E_x} - \frac{1}{E_z}$. In particular cases: m_1 = 0; l_1 = 0; n_1 = ± 1 l_1 = 0; n_1 = 0; m_1 = ±n_1 = 0; m_1 = 0; l_1 = ± 1.

Our solution (Gluhih et al., 2016) coincides with this one.

Conclusions

1) The found mathematical relationship between the elastic constants in the principal directions of anisotropy simplifies the solution of elastic problems for elastic fiber composite materials.

2) Experimental determination of elastic constants can be streamlined.

3) Using the suggested solution, with account for the tensorial nature of elasticity and strength characteristics, it will be possible to study the strength of composite materials considering the angle of the reinforcing fibers.

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ЗАДАЧА АНИЗОТРОПИИ УПРУГОСТИ И ПРОЧНОСТИ ВОЛОКНИСТЫХ АНИЗОТРОПНЫХ МАТЕРИАЛОВ

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Аннотация
В статье приведены новые материалы исследований анизотропии волокнистых материалов с цилиндрической анизотропией, к которым относятся армированные различными волокнами намоточные композиционные материалы. **Методы:** Автор дает математическое решение дифференциального уравнения четвёртого порядка в частных производных с двумя переменными для анизотропного ортотропного тела в полярных координатах. Для решения этого уравнения перевели в декартовы координаты и использовали функцию напряжений в виде суммы полиномов. **Результаты и обсуждения:** В итоге решения были получены два соотношения между постоянными упругости в главных направлениях анизотропии — так называемые параметры упругости. Одно из них получено впервые, а второе вытекает из решения задачи анизотропии криволинейного ортотропного тела С. Г. Лехницким, и полученное решение ему не противоречит. Таким образом, автор считает, что ортотропные материалы могут быть разделены на две группы. В одной группе, при переходе от радиального направления к тангенциальному, постоянные упругости принимают экстремальные значения при расположении слоёв под углами 0°, 60° и 90°. В другой, промежуточное экстремальное значение отсутствует, и постоянные упругости принимают экстремальные значения при наклоне слоёв под углами 0° и 90°. Результаты исследований представляют интерес при разработке новых высокопрочных композиционных материалов, при разработке новых технологий проектирования и изготовления строительных конструкций, высокопрочных конструкций из синтетических композиционных материалов.

Ключевые слова
Анизотропия свойств, композиционные материалы, математическая модель, цилиндрически анизотропное тело, модуль упругости, главные напряжения, постоянные упругости, коэффициент Пуассона, модуль сдвига.