DEFORMABILITY AND STABILITY OF RECTANGULAR SANDWICH PANELS WITH CUTS UNDER IN-PLANE LOADING

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Abstract

In this work the method of defining the critical load for rectangular sandwich panels is described having within themselves definite length cuts in a direction parallel with one of their sides. The coefficients of stability are determined by equating to zero the bending moments and the shear forces at the borders of the cut. This condition together with the rest of the boundary conditions for a panel forms the system of homogeneous equations. By equating to zero the determinant the critical load is obtained

Keywords

cut; panel; hole; stability; generalized function; in-plane loading

1. Introduction

Thin-walled shell structures are applied in the various scopes of engineering: machine building, aircraft and shipbuilding. In the construction they are widespread as rational solutions of industrial, agricultural, trade, sport buildings coverings.

The most serious violations of regularities are cuts, apertures, holes and cracks. The various rigid inclinations as reinforcement bars also violate the regularity.

The stress concentration zones in the places of violation of regularity (end of rib, discrete constraints) make significant impact on load bearing ability and stability of thin-walled load bearing structures. At this, the known traditional analytical and numerical methods for study of mode of deformation of ribbed thin-walled structures are less effective.

In this regard originates the necessity of development of new effective methods of solution of mentioned class of tasks. Currently intensively developing theory of generalized, in particular, discontinuous impulse, functions drastically extent the possibilities of analysis of thin-walled structure with various violations of regularity.

The three-layered panel with light-weight filler and two outer load bearing layers would be widely applied in construction of residential and public buildings as prefabricated typical element, in that in accordance with destination of building would be various structural features and additions as additional constraints, apertures for arrangement of openings such as doors or windows.

The methodology of calculation of three-layered planar elements with cuts and holes on strength and stability is not very much developed. Currently, a simple and convenient methodology there is not available for obtaining engineering design formulae for the definition of critical compressive in-plane loading taking into account various structural features, including values and arrangement of the weak regions (cuts, holes etc.) mentioned above.

The significance of the proposed method of calculation of the stability of three-layered plates with cuts is underlined by its potential of being applied in the design of shell structures of up-to-date materials with increased requirements for their reliability as well as from the absence of reliable effective methods that could be efficiently applied in engineering calculations at design.

The modern advances of mathematics and mechanics provides the possibility to initiate studies of most complex problems related to research of the behavior of thin-walled structures having one or several cuts of arbitrary shape. This is possible due to the research efforts of famous scientists, with results of that are included in a number of published works, including monographs of A.N. Guz (1971), E.I. Grigoliuk (1973), G.N. Savin (1968), I.N. Preobrazhenski (1981), B.K. Mikhailov (1980) and others. However, the level of solution regarding the stability of such structural elements rather lags behind the requirements of practical applications.

This stated problem was earlier considered in a number of reviews; for example, of N.A. Allumae (1972), O.D. Oniashvili (1957, 1969), G.I. Janelidze (1948), N.A. Nash (1957), P.M. Naghdi (1972), L.M. Kurshin (1962), B.K. Mikhailov and G.O. Kipiani (1987), G.O. Kipiani (2013).

The construction of large-span buildings and implementation in practice of modern low-modulus materials with high strength characteristics leads to necessity of considering of large, in comparison with thickness, deflections at analysis of thin-walled structures.

The development of various fields of industry and construction is related with improvement of existing and creation of new thin-walled structures that includes shells, plates, rods, having reinforcements, breaks, apertures, cuts, point supports. Group of such singularities are called as discontinuous parameters (Mikhailov,1980; Vol'mir, 1967; Mikhaylov and Kipiani, 1996, 1988(1,2)).

Most natural method to increase the stiffness of shells – it is the arrangement of ribs. On the borders of holes, at increasing of local loadings is advisable to perform the reinforcement of spatial structures by definite length ribs. The methods of ribs fastening to skin have an impact on the deformation behavior of such structures (Mikhailov, Kipiani et al., 1989, 1991; Kipiani et al., 1995, 1992, 2008, 2012, 2013, 2014).

The singularities of geometrical and physical parameters in thin-walled structures cause essential concentrations of stresses and result in areas of crack origination or plastic deformation. The load bearing capacity of such panels in most cases is determined by the strength or buckling capacity of the stress concentration areas. The other kinds of violation of regularity are presented by breaks that occur in folded and multi-wave coverings. By their impact on stress state they are similar to ribs.

2. Basic mathematical formulation

In order to be able to consider a cut discontinuous functions are introduced in the geometrical relations of theory of thin three-layered bending plates with lightweight filler. This is achieved by assignment of displacement components as ratios (Fig. 1) (Savin, 1968; Mikhailov 1980):

$$u_1^* = u_1 - \Delta u_1 H_x H_{yy}; \quad v_1^* = v_1 - \Delta v_1 H_x H_{yy};$$
$$u_2^* = u_2 - \Delta u_2 H_x H_{yy}; \quad v_2^* = v_2 - \Delta v_2 H_x H_{yy};$$
$$w^* = w - \Delta w H_x H_{yy}; \quad \gamma_1^* = \gamma_1 - \Delta w \delta_x, \qquad (1)$$

where u_1, v_1 – are the in-plane displacements of points of median surface of upper load bearing layer within the plane of the panel; u_2, v_2 – are the same for lower layer; Δu_1, Δu_2, Δv_1, Δv_2 – are the divergence of edges on line of cut of median surfaces accordingly of upper and lower layers; Δw – is the out-of-plane displacement of edge on line of cut perpendicular to the plane of the panel**;** $\Delta\gamma_1$ – is the angle of break of median surface on line of cut; H_x=H(x-a_1), H(y-b_1), H(y-b_2) – are the Heaviside functions; H_{yy}=H(y-b_1)-(y-b_2) – are the special function consisting from unit functions; δ=δ(x-a_1) – is the delta-function of Dirac.

For obtaining the governing equations it is necessary to substitute the expressions of equation 1 by known ratios of elasticity for three-layered plates included in the equations of equilibrium of an infinite small element (Mikhailov, 1991). Towards this it is necessary to take into account that the values Δu_1, Δu_2, Δv_1, Δv_2, Δw and $\Delta\gamma_1$ are the regular functions of coordinate *y* within the cut and are equal at $y=y_1$ and $y=y_2$ as at the ends of the cut it is not possible to have stepwise changes of the displacement or of the angle of rotation because such changes will introduce breaks and discontinuities along the line of the cut. Next, proceeding with the necessary transformations it is possible to exclude the components of in-plane displacements, in a similar way as this is done in the theory of continuous plates, and thus reduce the system of equations to a single equation related to the function of the deflection (the out-of-plane displacement ***w***). This is stated by the following expression**.**

$$\left[2B\left(h+\frac{t}{2}\right)^2+2D\right]\nabla^4 w-\frac{Bh}{G_3}\nabla^6 w=$$
$$=\left(1-\frac{Bh}{G_1}\right)P+2\left[B\left(h+\frac{t}{2}\right)^2+2D\right]$$
$$\left[\Delta w\delta_x^{III}+2\Delta w_y^{II}\delta_x^{IV}+\Delta w_y^{IV}H_x\right)H_{yy}+$$
$$+\left(\Delta\gamma_1\delta_x^{II}+\Delta\gamma_{1y}^{II}\delta_x\right)H_{yy}+$$
$$+2\Delta w_y^{III}H_x\delta_{yy}+\Delta w_y^{II}H_x\delta_{yy}^{I}+\Delta\gamma_{1y}^{I}\delta_x\delta_{yy}-\frac{Bh}{G_3}\times$$
$$\times\left[\Delta w_y\delta_x^{VI}+2\Delta w_y^{II}\delta_x^{III}+\Delta w_y\delta_x^{I}+\Delta\gamma_1\delta_x^{IV}+\Delta\gamma_{1y}^{II}\delta_x^{II}\right]H_{yy}+$$

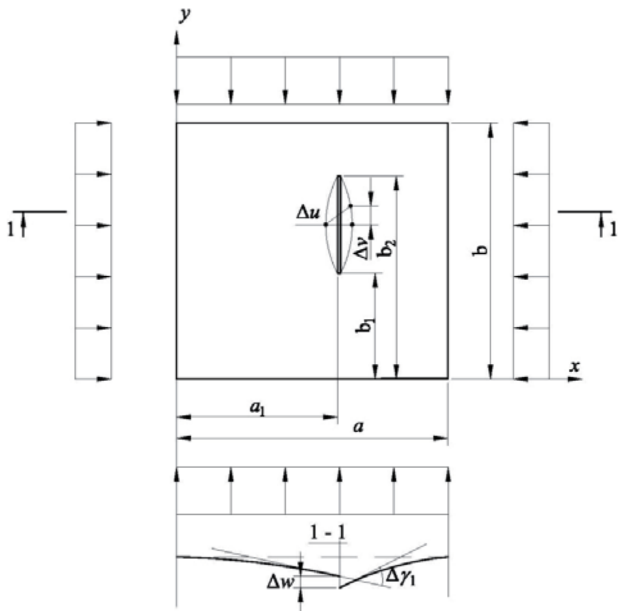

Fig. 1. Displacements Δu, Δ*v*, Δw and Δγ in upper and lower layers

$$+\left(\Delta w_y^{II}\delta_x^{III}+2\Delta w_y^{IV}\delta_x+\Delta w_y^{VI}H_x+\Delta\gamma_{1y}^{II}\delta_x^{II}+\Delta\gamma_{1y}^{IV}\delta_x\right)H_{yy}+$$
$$+\left(4\Delta w_y^{IV}\delta_x^{I}+2\Delta w_y^{V}H_x+2\Delta\gamma_{1y}^{III}\delta_x\right)\delta_{yy}+$$
$$+\left(\Delta w\delta_x'''+2\Delta w_y^{II}\delta_x^{I}+\Delta w_y^{IV}H_x\right)\delta_{yy}^{I}+$$
$$+2\Delta w_y^{III}\delta_x'\dot{\delta}_{yy}+\Delta w_y^{II}\delta_x^{I}\delta_{yy}^{I}+\Delta\gamma_{1y}^{I}\delta_x^{II}\delta_{yy}+$$
$$+2\Delta w_y^{V}H_x\delta_{yy}+\Delta w_y^{IV}H_x\delta_{yy}^{I}+2\Delta w_y^{III}H_x\delta_{yy}^{II}+$$
$$+2\Delta\gamma_{1y}^{II}\delta_x\delta_{yy}^{I}+2\Delta w_y^{III}H_x\delta_{yy}^{II}+\Delta w_y^{II}H_x\delta_{yy}'''+\Delta\gamma_{1y}^{I}\delta_x\delta_{yy}^{Iy} \quad (2)$$

As it is known, at study of stability it is advisable instead the external load P introduce the transverse fictitious load equal to summand of projections of compressive and shear forces on direction of normal to the non-deformed surface. All notifications are same as in (1).

$$P=P_f=T_1w_x^{II}+2Sw_{xy}^{II}+I_2w_y^{II} \quad (3)$$

If in equation (2) it is assumed that the shear modulus G_f (It is modulus of filler) tends to infinity, then the governing equation for a three-layered plate with a cut is reduced to the equation for single-layered plate with a cut; the bending stiffness of this sandwich plate is equal to $2\left[D+B\left(h+\frac{t}{2}\right)^2\right]$ and represents the stiffness of a section with two load bearing layers at distance of 2h from each other. Such equation was obtained in an earlier work [1].

The coefficients Δw and $\Delta\gamma_1$ are determined from conditions of equality to zero of moment and shear force on edges of cut. If only takes place break without divergence of edges, then would be accepted Δw=0, that commonly occurs at external compression on contour. Then the equation of stability will be as stated in previous work (Kipiani, 2008)

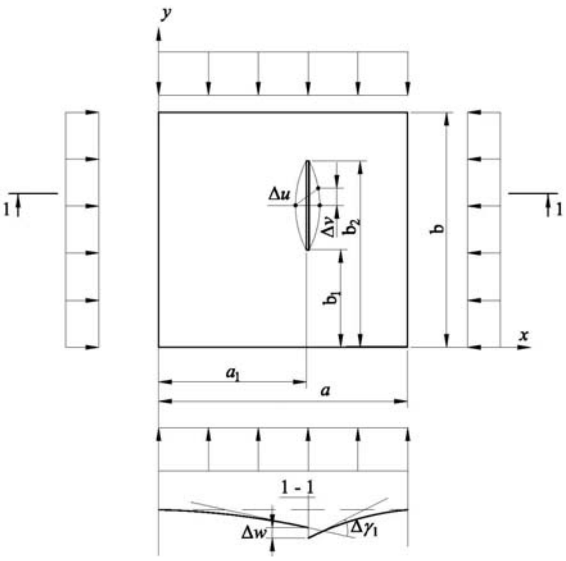

Fig. 2. Design diagram for compressed three-layered plate with cut

$$2\left[B\left(h+\frac{t}{2}\right)^2+2D\right]\nabla^4w-\frac{Bh}{G_3}\nabla^6w=$$
$$=\left(1-\frac{Bh}{G_1}\right)\left(T_1^0w_x^{II}+2S^\circ w_{xy}^{II}+T_2^0w_y^{II}\right)+$$
$$+2\left[B\left(h+\frac{t}{2}\right)^2+2D\right]\times$$
$$\times\left[\left(\Delta\gamma_1\delta_x^{II}+\Delta\gamma_{1y}^{II}\delta_x\right)H_{yy}+\Delta\gamma_{1y}^{I}\delta_x\delta_{yy}\right]-$$
$$-\frac{Bh}{G_f}\left[\left(\Delta\gamma_1\delta_x^{IV}+\Delta\gamma_{1y}^{II}\delta_x^{II}+\Delta\gamma_{1y}^{II}\delta_x^{II}+\Delta\gamma_{1y}^{IV}\delta_x\right)H_{yy}+\right.$$
$$+\left(2\Delta\gamma_{1y}^{III}\delta_x+\Delta\gamma_{1y}^{I}\delta_x^{II}+\Delta\gamma_{1y}^{III}\delta_x\right)\delta_{yy}+$$
$$\left.+2\Delta\gamma_{1y}^{II}\delta_x\delta_{yy}^{I}+\Delta\gamma_{1y}^{I}\delta_x\delta_{yy}^{II}\right]. \quad (4)$$

Let's assume that plate is compressed in direction perpendicular to line of cut, by load, uniformly distributed on two apposite edges x=0, x=a. Then $S^\circ=T_2=0$. If we assume that the edge conditions give the possibility in the first approximation represent the function $w_1\Delta\gamma_1$ as

$$w=w_1(x)\sin\beta_1y;\ \Delta\gamma=\Delta\gamma_{1(1)}\sin\bar{\beta}_1y, \quad (5)$$

where

$$\beta=\frac{\pi}{b};\ \bar{\beta}=\frac{\pi}{b'}.$$

Then the solution of equation (4) would be written down as

$$w=\left[w_1(x)+\Delta\gamma_{1(1)}f_1(x)\right]\sin\beta_1y, \quad (6)$$

where the function $f_1(x)$ has an discontinuous character that corresponds for distribution of displacement and angle of rotation and would be written down as

$$f_1(x)=2\left[B\left(h+\frac{t}{2}\right)^2+D\right]\left[\left(\psi_x''-\bar{\beta}_1^2\psi_x\right)a_1^0+\bar{\beta}_1\psi_xa_1\right]-$$
$$-\frac{Bh}{G_f}\left[\left(\psi_x^{1V}-2\bar{\beta}_1^2\psi_x''+\bar{\beta}_1^4\psi_x\right)a_1^0+\left(-2\bar{\beta}_1^3\psi_x+\bar{\beta}_1^3\psi_x\right)a_1-\right.$$
$$\left.-2\bar{\beta}_1^2\psi_xa_1'+\bar{\beta}_1\psi_xa_x''\right]. \quad (7)$$

The function $w_i(x_i)\psi_x''$ represents the solution of the relevant equations

$$2\left[B\left(h+\frac{t}{2}\right)^2+D\right]\left(\frac{d^2}{dx^2}-\beta_1^2\right)^2w_1(x)-$$
$$-\frac{Bh}{G_f}\left(\frac{d^2}{dx^2}-\beta_1^2\right)^3w_1(x)=\left(1-\frac{Bh}{G_f}\right)T_1^0w_x^{II};$$
$$2\left[B\left(h+\frac{t}{2}\right)^2+D\right]\left(\frac{d^2}{dx^2}-\beta_1^2\right)^2\psi_x''-$$
$$-\frac{Bh}{G_f}\left(\frac{d^2}{dx^2}-\beta_1^2\right)^3\psi_x''=\delta_x'', \quad (8)$$

$h=0,1,2,3,4.$

The function $w(x)$ would be presented as:

$$w(x)=w_1\sin\alpha x, \quad (9)$$

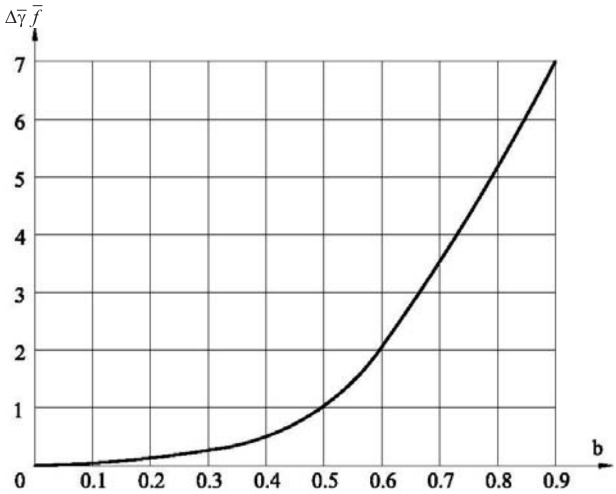

Fig. 3. Diagram of dependency of $\Delta\bar{\gamma}\,\bar{f}$ from length of cut

where

$$\alpha = \frac{\pi}{a}.$$

The coefficient $\Delta\gamma_{1(1)}$ would be determined from condition to equality to zero of moment on line x=x_1. This condition together with the boundary conditions on the edges x=0, x=a represent a system of simultaneous equations; next, the condition of equality to zero of their determinant forms the basis on which the critical loading is found.

As case let's consider the analysis on stability of square sandwich plate with symmetrically located cut, length of that is equal to one third of side of rectangular contour.

Results of calculation:

In the first approximation $T_{cr} = 0.42T^{\circ}_{cr}$

In the second approximation $T_{cr} = 0.687T^{\circ}_{cr}$

In the third approximation $T_{cr} = 0.774T^{\circ}_{cr}$

In the ninth approximation $T_{cr} = 0.933T^{\circ}_{cr}$

In the tenth approximation $T_{cr} = 0.935T^{\circ}_{cr}$

where T°_{cr} – is the critical loading on continuous sandwich plate.

As the second case is considered same dimensions plate. But the successive approximation process in this case is converged better. So in the first approximation we have

$$T_{cr} = 0.942T^{\circ}_{cr}$$

and in the second approximation $T_{cr} = 0.932T^{\circ}_{cr}$.

At this rather simply would been observed the dependency of coefficient $\Delta\bar{\gamma}\,\bar{f}$, and accordingly, reducing of critical loading on length of cut. The diagram of dependency of $\Delta\bar{\gamma}\,\bar{f}$ from length of cut is presented on Fig. 3.

As the second case is considered sandwich square plate with symmetrically located aperture. the length of that is equal to one third of plate external contour length.

The calculation gives the value of critical loading (in the second approximation)

$$T_{cr} = 0.715T^{\circ}_{cr}.$$

For comparison with respect of continuous plate we have

$$T_{cr} = 0.755T^{\circ}_{cr}.$$

By solution of corresponding static task, the process of successive approximation is convergence significantly better.

3. Conclusion

The basic mathematical formulations are presented that can be utilized towards the determination of the critical loading for a rectangular sandwich plate having a cut of finite length within such a plate parallel to one of its sides.

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