Introduction
The possibility to obtain the same wood strength upon compression along fibers and static bending was theoretically confirmed in experimental studies of German scientist H. Kubler. He proposed logarithmic distribution of internal forces over the tree stem volume.

Depending on the relative size of the core zone, the wood strength upon compression along fibers and static bending can be virtually the same at radii of the core zone $R_0 = (0.6-0.67)R$.

If the core zone size is more than 0.73R, the wood strength significantly increases upon compression along fibers.

Natural building materials of vegetable origin are formed when growing, following the direction of major stresses and strains. This provides the required strength and resilience (Glukhikh, Okhlopkova, 2017). Development of a high-strength fiber structure is mainly caused by critical external force effects, where the main external effect is a wind load. It facilitates development of a high-strength stem and a crown with a field of internal forces reducing bending stresses in the weakest (compressed) part of the stem and increasing those in a stronger (stretched) part of the stem.

Eventually, due to stresses established in the stem and roots, the risk of stem breaking or falling together with the roots under a critical wind load decreases during tree growth (Glukhikh, 2017).

According to the data (Kubler, 1959), stresses arising under critical wind loads and internal forces in the tree stem, formed in response to such stresses, reach their limit values.

Sawn-timber pieces sawn out of such tree stems and structural blanks for wooden building structures are stressed as the tree stem wood. Due to appearance of internal forces, the load-bearing capacity of wooden structures decreases.

The existing methods of stress grading of structural sawn timber do not allow revealing internal forces, although some publications H. Kubler, A. Ilien,
Kollmann, Kuffner state their effect on deflection and, consequently, on the modulus of elasticity and associated bending strength of sawn timber. Some works (Glukhikh, Okhlopkova, 2017) state that the arrangement of annual layers affects the lengthwise bending strength of sawn timber. However, the real reason of wood strength decrease (internal forces formed during tree growth) is not considered. Wood refers to materials actively responding to changes in external effects during growth, according to the "bionic principles of adjusting parameters of a stress-strain state in structures" (Glukhikh et al., 2017).

When external effects change, the inflow of nutrients to the weakest parts of the stem, wood density, modulus of elasticity, and thickness of cell walls increase. This leads to wood strength increase. The process continues during the whole period of tree growth.

In the work (Kuznetsov, 1950), the wood strength formation upon compression along fibers and static bending for some cases of internal forces' distribution along the volume of the tree stem (for trees growing in Europe, Canada, North and South America, South Asia and Russia) is theoretically justified. The available ratios of wood ultimate strengths upon compression along fibers and static bending (according to V. N. Volyansky) confirm our theoretical assumptions. This allows predicting the wood strength upon compression along fibers, depending on the stem taper, size of the core and sap zones, and wood species.

Methods of study

It is known that the relative size of the core zone increases along the height of the tree stem from the butt to the top due to the reduction in the outer diameter. Based on the important studies, the structural tree stem model can be represented as a rod of uniform strength, rigidly fixed in the butt portion. This means that axial stresses in peripheral fibers do not change along the height of the tree stem. According to our studies (Glukhikh, Okhlopkova, 2017), the maximum compression stress changes along the height of the tree stem.

According to the basic principles of bionics, it can be assumed that the wood strength changes from the butt to the top upon compression along fibers.

If, for example, the size of the core zone $R_0 = 0.86 \cdot R$ (which corresponds to the distribution of internal forces according to the law of the fourteenth-degree paraboloid), then the ratio of wood ultimate strengths upon static bending and compression along fibers is 2.24 (Rodionov et al., 1956).

If the size of the core zone $R_0 = 0.707R$ (which corresponds to the distribution of internal forces according to the law of the second-degree paraboloid), then the ratio of wood ultimate strengths upon static bending and compression along fibers is 2.24 (Rodionov et al., 1956).

If we denote the radius of the top end as $R_0$, then the radius of the log butt end will be equal to:

$$R_k = R_0 + K_c \cdot L$$

where the log taper coefficient is $K_c = \frac{R_k - R_b}{L - \log \text{length}}$.

Considering uniform strength of wood along the length of the log, the following equation can be developed for each end section according to equation (1):

$$k_{n(b)} \cdot R_{n(b)}^n = k_{n(k)} \cdot R_{n(k)}^n$$

whence it is possible to calculate the radius of the butt-end core zone given the known radius of the top-end core zone, for example:

$$R_{n(k)}^n = R_{n(b)}^n \cdot \frac{k_{n(k)}}{k_{n(b)}}$$
Results of studies

If we assume that internal stresses at both ends are distributed according to the law of the second-degree paraboloid, then the radius of the top core zone will be as follows (given the log length L = 6 m, $R_k = 20$ cm; $R_s = 14$ cm):

$$ R_{0(k)} = R_{0(b)} \frac{R_k^2}{R_s^2} = 0.707 \cdot R_s \cdot \frac{14}{20^2} \cdot R_b = 0.505 R_b $$

Under different laws of internal forces' distribution at the ends, it is possible to confirm the validity of the accepted hypotheses.

For example, if the radius of the log top-end core zone is $0.707 R_b$ at $R_s = 14$ cm and the radius of the butt-end core zone is $0.862 R_s$ at $R_k = 20$ cm, we will obtain the following based on (8):

$$ \begin{align*}
R_{0(b)} &= 0.707 \cdot 14 = 0.707 \cdot 14 \\
R_{0(k)} &= 0.862 \cdot 20 = 0.862 \cdot 20 = 0.5741
\end{align*} $$

The following ratio should equal this value:

$$ k_{n(k)} = \frac{7 \sigma_o + \sigma_0}{\sigma_o + \sigma_0} \cdot \frac{R_{0(k)}^2}{R_{0(b)}^2} $$

Let us provide calculation for determining the ratio of the wood ultimate strengths upon static bending and compression along fibers as an example (Table 1).

### Table 1. Calculated values of ultimate strengths upon static bending $\sigma_k$ and compression along fibers $\sigma_{bc}$

<table>
<thead>
<tr>
<th>Radius of the core zone at the log end</th>
<th>Top end</th>
<th>Butt end</th>
</tr>
</thead>
<tbody>
<tr>
<td>at $R_b = 100$ mm, $R_k = 150$ mm</td>
<td>$R_{0(b)} = 0.707 R_b = 70.7$ mm</td>
<td>$R_{0(k)} = 0.818 R_k = 122.7$ mm</td>
</tr>
<tr>
<td>Sapwood width, mm</td>
<td>$S_b = 100 - 70.7 = 29.3$ mm</td>
<td>$S_k = 150 - 122.7 = 27.3$ mm</td>
</tr>
<tr>
<td>Ratio of ultimate strengths</td>
<td>$\left( \frac{\sigma_{BN}}{\sigma_{BC}} \right)_b = 1.778$</td>
<td>$\left( \frac{\sigma_{BN}}{\sigma_{BC}} \right)_k = 2.273$</td>
</tr>
<tr>
<td>Ultimate strength upon compression</td>
<td>$\sigma_{BC(b)} = 0.5624 \cdot \sigma_{BN}$</td>
<td>$\sigma_{BC(k)} = 0.44 \cdot \sigma_{BN}$</td>
</tr>
<tr>
<td>Along fibers</td>
<td>$\left( \frac{\sigma_{BC}}{\sigma_{BC}} \right)_b = 0.5624$</td>
<td>$\left( \frac{\sigma_{BC}}{\sigma_{BC}} \right)_k = 0.44 \cdot 1.278$</td>
</tr>
</tbody>
</table>

The results of calculating the ratios of wood ultimate strengths upon compression along fibers at the top end and butt end of logs at different relative sizes of the core zone are given in Table 2.

### Table 2. Changes in the wood strength upon compression along fibers along the length of sawn timber

<table>
<thead>
<tr>
<th>Relative size of the core zone at the log ends</th>
<th>Wood strength upon compression along fibers at the ends (in relative units)</th>
<th>Ratio of ultimate strengths upon compression along fibers at butt and top ends of the board</th>
</tr>
</thead>
<tbody>
<tr>
<td>Butt Top</td>
<td>Butt Top</td>
<td>Butt Top</td>
</tr>
<tr>
<td>$0.862 R_s$</td>
<td>$0.667 R_s$</td>
<td>2.24</td>
</tr>
<tr>
<td>$0.76 R_s$</td>
<td>$0.667 R_s$</td>
<td>2.191</td>
</tr>
<tr>
<td>$0.707 R_s$</td>
<td>$0.606 R_s$</td>
<td>1.778</td>
</tr>
<tr>
<td>$0.731 R_s$</td>
<td>$0.666 R_s$</td>
<td>1.524</td>
</tr>
<tr>
<td>$0.818 R_s$</td>
<td>$0.707 R_s$</td>
<td>1.778</td>
</tr>
<tr>
<td>$0.794 R_s$</td>
<td>$0.76 R_s$</td>
<td>2.265</td>
</tr>
<tr>
<td>$0.85 R_s$</td>
<td>$0.867 R_s$</td>
<td>2.273</td>
</tr>
<tr>
<td>$0.862 R_s$</td>
<td>$0.794 R_s$</td>
<td>2.24</td>
</tr>
</tbody>
</table>

Results

Depending on the size of the core zone, the wood strength upon compression along fibers varies from the butt end to the top end, given the same strength upon static bending. Moreover, if the relative size of the core zone decreases, the wood strength upon compression along the fibers decreases as well, which corresponds to the wood at the top end.

Given a slight difference in the size of the core zone at the ends of the log, the wood strength upon compression along fibers changes insignificantly (1–3%) along the length of the board. If the size of the core zone increases by 25%, the wood strength upon compression along fibers decreases 2.191 times. If the difference in the size of the core zone at the ends of the logs is 42%, the wood strength decreases 2.24 times compared to the wood strength upon static bending.

If sizes of the core zone at the ends of the log are, for example, $R_{0(b)} = 0.707 R_b$ and $R_{0(k)} = 0.766 R_b$, the ratios of ultimate strengths upon static bending and compression along fibers will be 1.778 and 2.191, respectively.

Based on those ratios, it is possible to calculate the wood ultimate strength upon compression along fibers at the butt and top ends, respectively.

On the basis of the foregoing, the experimental values of internal forces, obtained by German scientist H. Kubler (Kubler, 1959), correspond to wood samples with the size of the core zone close to $R_s = 0.606 R$, which is illustrated in the published photographs. As for such size of the core zone, the logarithmic law of internal forces' distribution along the volume of the tree stem, accepted in Kubler's studies, can be considered as justified: the wood strength upon static bending and compression along fibers turns out to be the same, and it was proved by our theoretical studies.
References


Strikha, I.A. (1954). Prichina deformatsii detalei iz drevesyny i sposoby ee umensheniiia [The cause of deformation of wood components and ways to reduce it]. *Derevoobrabativaushaya promishlennost* [Woodworking industry], 7. (in Russian)